UNIVERSIDAD DE MURCIA

Escuela de Doctorado

TESIS DOCTORAL

Sistemas de D-branas y compactificaciones de cuerda abierta

D-brane systems and open string compactifications

AUTOR Juan Ramón Balaguer Tornel

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A mi madre Carmen, a Virginia y a mi hija Carmencita:
las tres cuerdas fundamentales que dan sentido a mi universo.

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Resumen en español

Esta tesis estudia diversos aspectos de la teoría de cuerdas, siendo el nexo principal la física de las D-branas, que constituyen objetos extendidos fundamentales sobre los que pueden terminar las cuerdas abiertas y juegan un papel central tanto en la dinámica no perturbativa como en la conexión con modelos efectivos de baja energía. La motivación del estudio de la teoría de cuerdas surge del desafío de reconciliar el Modelo Estándar de la física de partículas, que describe tres de las cuatro interacciones fundamentales en términos de una teoría cuántica de campos, con la Relatividad General de Einstein, que sigue siendo una teoría clásica de la gravedad. Esta tensión histórica apunta a la necesidad de una teoría cuántica de la gravedad consistente. La teoría de cuerdas, con su redefinición de los constituyentes fundamentales de la naturaleza como cuerdas vibrantes unidimensionales en lugar de partículas puntuales, aspira no solo a ser una teoría cuántica de la gravedad, sino también a unificar todas las interacciones fundamentales, lo que la convierte en una candidata natural a una teoría del todo. No obstante, la consistencia matemática de la teoría de cuerdas requiere la existencia de dimensiones espaciales adicionales más allá de las cuatro que experimentamos en nuestra vida cotidiana, y la física efectiva de nuestro universo depende de cómo se compactifican estas dimensiones extra. La compactificación no es, por tanto, un detalle incidental, sino una característica central: la geometría, la topología y los flujos de los espacios internos determinan la física de baja energía que observamos en cuatro dimensiones.

La tesis se estructura en torno a dos líneas principales de investigación. La primera explora aspectos de la holografía de precisión mediante la construcción de nuevas configuraciones de branas que proporcionan duales gravitatorios para teorías de campos de menor dimensión en el contexto de la correspondencia AdS/CFT. Esta correspondencia, uno de los desarrollos más influyentes de la física teórica moderna, propone una dualidad entre una teoría gravitatoria en un espacio anti-de Sitter (AdS) y una teoría de campos conforme (CFT) situada en su frontera. Aunque el ejemplo original y mejor entendido involucra la teoría de cuerdas tipo IIB en $\mathrm{AdS}_5 \times S^5$ siendo dual a la teoría de super Yang-Mills (SYM) $\mathcal{N} = 4$ en cuatro dimensiones, la extensión a casos con menos simetría o en dimensiones inferiores sigue siendo un reto. Una de las dificultades principales radica en encontrar configuraciones explícitas de D-branas cuyo límite de horizonte cercano dé lugar a las geometrías AdS deseadas. Al investigar sistemas de branas en teoría de cuerdas tipo IIB, este trabajo propone y analiza nuevas soluciones que dan lugar a geometrías AdS_3. Estas construcciones ofrecen realizaciones explícitas en teoría de cuerdas de deformaciones de SYM bidimensionales.

La segunda línea de investigación se centra en las compactificaciones con flujos en presencia de D-branas y planos orientifold, considerando las excitaciones de cuerda abierta. Los escenarios tradicionales de compactificación en teoría de cuerdas consideraban principalmente sectores de cuerda cerrada, pero la inclusión de cuerdas abiertas, ligadas a las D-branas, enriquece la estructura de las teorías efectivas resultantes. Las excitaciones de cuerdas abiertas dan lugar a campos gauge asociados a simetrías locales, así como a moduli dinámicos que describen las posiciones de las branas, mientras que la presencia de estas introduce flujos no triviales que modifican el potencial escalar de la acción efectiva. Un análisis cuidadoso de estas contribuciones es esencial para lograr modelos consistentes en dimensiones inferiores con moduli estabilizados. En este marco, la tesis desarrolla una correspondencia entre las reducciones en variedades de grupo con orientifold de la tipo IIB a seis dimensiones y las teorías de supergravedad gaugeadas con $\mathcal{N} = (1, 1)$, demostrando una coincidencia exacta de los potenciales escalares. Esto incluye contribuciones no sólo del sector de cuerda cerrada, sino también del sector de cuerda abierta, incluyendo grupos gauge no abelianos y flujos internos de Yang-Mills. El resultado proporciona un ejemplo de construcciones top-down provenientes de la teoría de cuerdas y clasificaciones bottom-up de supergravedades gaugeadas ampliando análisis anteriores que se habían centrado exclusivamente en el sector de cuerda cerrada.

La tesis consta de siete capítulos. El primero introduce el marco conceptual de la tesis y presenta de manera sintética las dos grandes líneas de trabajo que se desarrollan en ella: el estudio de la holografía y el análisis de compactificaciones con flujos desde una perspectiva efectiva en supergravedad.

En el segundo capítulo se revisan los elementos fundamentales de la teoría de cuerdas relevantes para el análisis posterior. La formulación del modelo sigma no lineal mediante la acción de Polyakov proporciona la base para describir la propagación de cuerdas en "backgrounds" curvos. La invariancia conforme, codificada en la anulación de las funciones beta, en el régimen perturbativo, conduce a ecuaciones que, a primer orden en α', reproducen las ecuaciones de movimiento de las teorías efectivas de supergravedad en diez dimensiones, estableciendo así un vínculo entre la teoría bidimensional de la hoja de mundo y la descripción efectiva de los backgrounds en el régimen de baja energía. Además, se revisa el papel esencial de las dualidades (en particular la T-dualidad y la S-dualidad) al relacionar distintas teorías de cuerdas y conectar regímenes de acoplamiento débil y fuerte. Estas dualidades establecen que lo que en apariencia son teorías distintas son, en realidad, diferentes límites de un marco unificado.

Posteriormente, en el tercer capítulo se revisan los principales objetos cargados de la teoría de cuerdas, en particular las D-branas y los planos orientifold. Se introducen las acciones efectivas que describen las D-branas y los O-planos, y se analiza su interacción en sistemas Op/Dp. A continuación, se presentan las soluciones de supergravedad asociadas a las Dp-branas, la cuerda fundamental y la NS5-brana, que actúan como "backgrounds" clásicos generados por estos objetos. Sobre esta base, se introduce la correspondencia AdS/CFT, destacando el papel de las branas en la holografía. Finalmente, el capítulo concluye con una discusión sobre los proyectores de supersimetría en la teoría tipo IIB, que proporcionan un método sistemático para caracterizar la supersimetría preservada por distintas configuraciones de branas.

El capítulo cuarto marca el final de la revisión bibliográfica y está dedicado al estudio de los gaugeos en supergravedad y de su origen en dimensiones superiores. Tras revisar la estructura de las teorías de supergravedad no gaugeadas, con especial atención a sus variedades escalares en dimensiones inferiores, se introduce el formalismo del tensor de embedding como una herramienta poderosa para describir supergravedades gaugeadas. Se explica el papel de los desplazamientos fermiónicos y su relación con las soluciones de vacío, seguido de un análisis detallado de los potenciales escalares resultantes, incluidas las simplificaciones que se obtienen en el origen de la variedad escalar. A continuación, el capítulo explora el origen de estos gaugeos en dimensiones superiores, abarcando compactificaciones en variedades de grupo, reducciones del sector bulk de las teorías de tipo II, y la aparición de tadpoles y mecanismos de Green–Schwarz. Todo ello sienta las bases para el posterior análisis de las compactificaciones con flujos y su relación con la supergravedad gaugeada.

Llegados a este punto, se presentan las contribuciones originales en los capítulos quinto y sexto. El quinto capítulo está basado en [57] y trata sobre la holografía de precisión. En él se estudian intersecciones de branas D3–D5–D7 y la forma general de las soluciones dentro de esta clase. Tras argumentar la ausencia de geometrías AdS_3 de tipo horizonte cercano en este sistema, se discute brevemente la interpretación de estas soluciones como perfiles supersimétricos dependientes de la posición para el acoplamiento de Yang-Mills sobre las branas D3, debidos a la presencia de branas de defecto D5–D7. A continuación, se analizan las intersecciones D3–D3–D7, donde se muestra cómo, al tomar el límite near-horizon, aparece una geometría de tipo $\mathrm{AdS}_3 \times S^3 \times \mathbb{T}^2$ envuelta sobre una superficie de Riemann Σ. Finalmente, se considera una configuración más elaborada que preserva la misma simetría del espacio-tiempo y la misma cantidad de supersimetría que los casos anteriores, en la que intervienen dos D5 y dos NS5 colocadas en disposiciones distintas, junto con branas D7. En este caso, las soluciones son de tipo $\mathrm{AdS}_3 \times S^2$ envueltas sobre $\mathbb{T}^3 \times \Sigma$.

En el capítulo sexto, basado en [58], se amplía el análisis de las supergravedades gaugeadas en seis dimensiones al incluir contribuciones de cuerda abierta. Se estudian en detalle las reducciones orientifold de la teoría de cuerdas tipo IIB en presencia de sistemas O5/D5, O7/D7 y O9/D9. La novedad de este análisis reside en la incorporación de moduli dinámicos de posición de brana y grupos gauge no abelianos con flujos asociados de Yang-Mills. Aunque estos ingredientes son técnicamente complejos desde un enfoque top-down, pueden acomodarse de manera natural en la clasificación bottom-up de supergravedades gaugeadas mediante la introducción de multipletes vectoriales adicionales. Un resultado clave de este trabajo es la demostración de que el potencial escalar obtenido de la acción efectiva de reducciones orientifold de tipo IIB, incluyendo contribuciones de cuerdas abiertas, coincide de manera precisa con el potencial escalar derivado de la supergravedad gaugeada con $\mathcal{N} = (1, 1)$ y el número adecuado de multipletes vectoriales. Esta correspondencia altamente no trivial confirma la consistencia del enfoque y proporciona una realización concreta del principio de universalidad de cuerdas en seis dimensiones. Además, el análisis revela modificaciones de los “field strengths” del “bulk” originadas por los campos vectoriales de cuerda abierta, análogas al mecanismo de Green-Schwarz en la heterótica. Estas modificaciones, interpretadas como versiones U-duales de los términos de Green-Schwarz.

La tesis enfatiza que tales efectos van más allá de un mero ejercicio matemático. Señalan la necesidad física de incluir correcciones en α' en cualquier descripción realista de compactificaciones con flujos y cuerdas abiertas. Aunque la consistencia de la coincidencia de los potenciales escalares se establece a α' finito, la cuestión más amplia de la fiabilidad física permanece abierta y constituye una línea de investigación futura. Sin embargo, los resultados logrados representan ya un paso importante en la construcción de un puente entre compactificaciones explícitas en teoría de cuerdas y descripciones efectivas en supergravedad.

En conjunto, las dos líneas de investigación desarrolladas en esta tesis subrayan el papel central de las D-branas en la teoría de cuerdas. En el lado holográfico, las construcciones con branas proporcionan el origen microscópico de los vacíos AdS y sus teorías conformes duales, permitiendo pruebas de precisión de la correspondencia AdS/CFT y sus generalizaciones a CFTs con defectos. En el lado de la compactificación, las D-branas introducen excitaciones de cuerda abierta y flujos que amplían el paisaje de teorías efectivas en dimensiones inferiores, enriqueciendo el espectro de vacíos y conectando los escenarios de la teoría de cuerdas con las clasificaciones de supergravedades gaugeadas. Ambos aspectos convergen en la idea de que las D-branas son ingredientes fundamentales de la teoría de cuerdas, configurando tanto sus dualidades holográficas como sus escenarios de compactificación.

Las conclusiones de la tesis resumen los logros principales en el capítulo séptimo. En primer lugar, se identifican nuevas configuraciones de intersección de branas en teoría de cuerdas tipo IIB que generan geometrías AdS_3 en sus límites de horizonte cercano, proporcionando así realizaciones explícitas en branas de ciertas teorías conformes. En segundo lugar, se establece un diccionario sistemático entre reducciones orientifold de tipo IIB y supergravedades gaugeadas en seis dimensiones, incorporando no solo efectos de cuerda cerrada sino también de cuerda abierta. La coincidencia exacta de los potenciales escalares en ambas descripciones demuestra la solidez del enfoque. Estos logros no sólo profundizan nuestra comprensión de la estructura de los vacíos en teoría de cuerdas, sino que también abren direcciones prometedoras para investigaciones futuras, particularmente en lo que respecta al papel de las correcciones en α' y la viabilidad física de las compactificaciones con flujos inducidos por cuerdas abiertas.

En resumen, la tesis contribuye al esfuerzo más amplio de comprender el paisaje de la teoría de cuerdas al abordar dos problemas complementarios: la holografía de precisión y las compactificaciones con flujos de cuerdas abiertas. A través de construcciones explícitas de branas y análisis detallados de teorías efectivas, aclara cómo las D-branas configuran tanto los aspectos holográficos como los de compactificación de la teoría de cuerdas. Los resultados aquí presentados refuerzan así el estatus de las D-branas como bloques constructivos indispensables de la teoría de cuerdas y ponen de relieve su papel como hilo conductor entre holografía y compactificación.

Publications

List of publications resulting from the research carried out during the PhD:

- J. R. Balaguer, G. Dibitetto, and J. J. Fernández-Melgarejo, *New IIB intersecting brane solutions yielding supersymmetric* AdS_3 *vacua*, `arXiv:2104.03970` [hep-th], *JHEP 07 (2021) 134*.
- J. R. Balaguer, G. Dibitetto, J. J. Fernández-Melgarejo, and A. Ruipérez, *Open strings in IIB orientifold reductions*, `arXiv:2303.02040` [hep-th], *JHEP 07 (2023) 102*.
- J. R. Balaguer, V. Bevilacqua, G. Dibitetto, J. J. Fernández-Melgarejo, and G. Sudano, *Massive IIA flux compactifications with dynamical open strings*, `arXiv:2406.15310` [hep-th], *JHEP 03 (2025) 159*.

Only the results corresponding to the first two publications are included in this PhD thesis.

Contents

Chapter 1

Introduction

The twentieth century culminated in a remarkably precise understanding of the fundamental forces of Nature. The Standard Model of particle physics [1], validated by experiments at the LHC and other accelerators, describes the electromagnetic, weak, and strong interactions with stunning accuracy. Simultaneously, Einstein's General Theory of Relativity has passed every experimental test, from the precession of Mercury's orbit to the recent direct detection of gravitational waves by observatories like LIGO and Virgo [2]. These waves, ripples in the fabric of spacetime caused by cataclysmic events such as merging black holes, provide a spectacular confirmation of the classical dynamics of gravity. Yet, this very success highlights a profound theoretical schism: while the Standard Model is a quantum field theory, General Relativity remains a stubbornly classical description of gravity. The search for a unified framework that consistently describes all fundamental interactions, including gravity, within a quantum-mechanical setting stands as the paramount challenge in theoretical physics.

This challenge is not merely philosophical; it becomes pressing in extreme environments where strong gravity and quantum effects are inseparable, such as the singularities inside black holes or the first moments after the Big Bang. String theory has emerged as a leading candidate for such a unification. Its core premise is that the fundamental constituents of reality are not point-like particles but tiny, vibrating one-dimensional strings. This simple shift in perspective has profound consequences, one of which is the necessity of extra spatial dimensions for the mathematical consistency of the theory. The specific configuration and dynamics of these extra dimensions determine the effective laws of physic –the particle content, coupling constants, and the nature of spacetime itself– in our observable four-dimensional universe. Therefore, a central task in string theory is to classify and understand the stable, low-energy configurations, or vacua, that arise from the compactification of these extra dimensions.

The journey from ten-dimensional string theory to a four-dimensional universe involves the process of compactification. Early models used static, Ricci-flat internal spaces, but these typically led to theories with massless scalar fields, or moduli, whose values were not fixed. This is phenomenologically problematic, as moduli would mediate unobserved long-range forces. The mechanism of flux compactifications resolves this by turning on background "fluxes" (generalized magnetic fields). These fluxes warp the geometry and generate a potential that can stabilize the moduli, lifting the degeneracy and selecting specific vacua. This mechanism unlocks a rich landscape of possibilities, where the interplay between fluxes and geometry can produce four-dimensional spacetimes with different cosmological constants, each with distinct physical implications.

In this thesis we will address two central problems within the framework of string theory, both of which rely critically on the physics of D-branes. On the one hand, we will explore aspects of precision holography, aiming to test and refine the AdS/CFT correspondence by examining regimes where quantitative control can be achieved. On the other hand, we will investigate flux compactifications in the presence of Dp-branes and orientifold Op-planes. In this context, the inclusion of D-branes naturally introduces open-string degrees of freedom and their associated fluxes, thereby enriching the structure of the scalar potential and leading to a broader landscape of possible vacua. Together, these two lines of inquiry highlight the pivotal role of D-branes both in holography and in the dynamics of flux compactifications.

Precision Holography. Flux compactifications aim at studying the physical properties of all string theory backgrounds featuring lower-dimensional maximally symmetric geometries. Depending on whether the spacetime curvature is zero, positive, or negative, the corresponding backgrounds represent important playgrounds for string phenomenology. In particular, dS vacua may be used in order to produce toy models for late-time cosmology, while Mkw vacua provide effective models of gravity coupled to (non-)supersymmetric field theories and hence can be very valuable in order to study mechanisms like SUSY breaking. On the other hand, AdS vacua acquired great relevance ever since the very birth of the AdS/CFT correspondence.

Focusing on AdS in particular, an enormous technological progress has been made over the last two decades in the search for vacua preserving some residual supersymmetry. The crucial tool that has been widely employed in this context is the so-called pure spinor formalism [3, 4, 5]. It is based on the crucial interplay between background fluxes and geometry whenever SUSY is at least partially preserved in the desired vacuum. This has already allowed us to map out large portions of the string landscape of supersymmetric AdS vacua in various dimensions, revealing interesting structures and providing us with many successful holographic checks, which eventually taught us many things about the AdS/CFT correspondence.

However, an important missing piece in order to go beyond testing holography and put it on more solid grounds is the existence of a brane construction for a given AdS vacuum. The reason for this is that one may appeal to the aforementioned brane construction in order to constructively map the two sides of the correspondence into each other. This is precisely what was successfully carried out in the original work of [6] stating the correspondence between type IIB on $\mathrm{AdS}_5 \times S^5$ and $\mathcal{N} = 4$ SYM_4. It may be worth mentioning that, besides a few other examples, the vast majority of known SUSY AdS vacua do not yet admit a known brane picture. The main reason for this is that, the lower the amount of preserved SUSY, the more challenging it gets to find one.

Another advantage of possessing the brane description of a given vacuum is that it gives us the opportunity to study holography even beyond AdS space and hence beyond conformal theories. The first example of this is the so-called DW/QFT correspondence, which was proposed in [7]. This correspondence relates an asymptotically AdS domain wall (DW) to the RG flow of a QFT admitting a conformal fixed point. A crucially different physical situation of this type is that of having an asymptotically AdS *curved* DW (*e.g.* with lower-dimensional AdS slices). Such geometries were proposed in [8, 9] to provide a holographic description of defect CFTs (see also [10]). This was originally illustrated in the context of Janus solutions describing a conformal interface within $\mathcal{N} = 4$ SYM engineered by means of a position-dependent profile for the gauge coupling. On the other hand, a hint towards explaining conformal defects from branes had already been given in [11, 12], where it was shown how lower-dimensional AdS interfaces occur in the probe limit when placing defect branes within the background of some *mother* branes admitting a higher-dimensional AdS geometry in the near-horizon limit. A more explicit relation between these two descriptions of conformal defects was further shown in, *e.g.* [13, 14, 15, 16, 17, 18], though in the different context of 5d and 6d SCFTs.

A stack of D3 branes has (non-Abelian) $\mathcal{N} = 4$ SYM_4 as a worldvolume field theory description. This theory enjoys conformal symmetry and maximal rigid supersymmetry. On the supergravity side, an $\text{AdS}_5 \times S^5$ geometry emerges when taking the near-horizon limit of the D3 brane metric. By placing additional orthogonal branes on a stack of D3 branes, $(1+3)$D conformal symmetry is broken and one expects a lower-dimensional field theory description to emerge [19, 20, 10, 21]. In [22, 23], partially SUSY preserving deformations of $\mathcal{N} = 4$ SYM_4 with 2-dimensional spacetime dependent profiles for the couplings were studied. It was argued that their stringy origin is the presence of *defect branes* in the background. Consequently, a brane system with D3 branes containing an AdS_3 when taking the near horizon limit is expected as a holographic description of such field theories.

In this context, the aim of this thesis is to provide novel brane constructions within a particular setup in type IIB string theory that allow for the supergravity side description of the aforementioned SYM deformations. That is to say, we explore a set of brane intersections (i) containing at least one D3 brane, (ii) with no isometry directions along the orthogonal directions of the D3 brane and (iii) admitting AdS_3 vacua (see *e.g.*[24, 25, 26] for wide classifications of well-known classes of these vacua). In particular, some of these vacua preserving $\mathcal{N} = (4, 0)$ SUSY are related to (massive) IIA via a T-duality. These latter backgrounds were extensively studied in [27, 28, 29, 30, 31, 32]. We shall rely on the technique developed in [33] for constructing supergravity solutions describing *semilocalized* brane intersections, *i.e.* where the different brane charge distributions no longer obey the harmonic superposition principle. Such a tool was used in [34] and [35] to engineer AdS vacua with a clear underlying brane interpretation.

Open string flux compactification. Extracting consistent low energy effective descriptions from string theory is one the main challenges of theoretical high energy physics. Typically this procedure involves dimensional reduction and (partial) supersymmetry breaking. Depending on the mechanism used in order to realize them, a plethora of viable lower dimensional models arises, with varying amounts of supersymmetry in different dimensions. Low energy effective models obtained in this way are by construction UV consistent and belong to the string landscape.

On the other hand, by adopting a *bottom-up* approach instead, one could study different lower dimensional (supersymmetric) constructions and assess whether or not they can be consistently coupled with quantum gravity in a UV regime. This is the perspective promoted by the so-called Swampland Program [36, 37], which aims at identifying a set of consistency requirements that any effective theory must comply with, in order for it to admit a UV completion.

By restricting oneself to theories enjoying extended supersymmetry, the range of possibilities gets drastically reduced, up to the extent that UV consistency requirements in some instances may be even exhaustively analyzed. This certainly applies to the case of maximal supersymmetry, *i.e.* 32 supercharges. In 10D, the only consistent maximal supergravities are type IIA and type IIB supergravities and they exactly coincide with the low energy limits of the corresponding superstring theories, respectively. This may be viewed as a prime manifestation of *string universality.*

Now, still within 10D one may consider theories with half-maximal supersymmetry. In such a situation, the aforementioned universality principle was verified in [38] by showing that the only UV consistent half-maximal theories are $\mathcal{N} = 1$ supergravities with gauge groups given by either SO(32) or $E_8 \times E_8$. Those are indeed the only gauge symmetries that may be ever obtained by considering the low energy limits of heterotic or type I superstring theories.

In the last few decades we have learned a number of things concerning string theories with 16 supercharges and this allowed us to address the string universality issue in dimension lower than 10. By now we can consider it to be fully verified up to dimension 8 [39, 40, 41]. Besides, there have been recent developments even in dimension 7 and 6, the latter both with $(2, 0)$ [42] and $(1, 1)$ [43] supersymmetry, as well as some preliminary studies on $D < 6$ cases [44, 45, 46].

Our present work is to be placed within such a context, from which it draws its main motivations. We aim at building a bridge between top-down string theory constructions yielding 6D theories with $(1, 1)$ supersymmetry and the corresponding (gauged) supergravities, which are classified by means of bottom-up based organizing principles[1]. In more concrete terms, the stringy setup's relevant here are compactifications of type I/heterotic strings on $\mathbb{T}^4$, as well as orientifold reductions of type IIA/IIB on $\mathbb{T}^4$, or M theory on $\mathbb{T}^5$. Our interest towards $(1, 1)$ supergravity rather than for the $(2, 0)$ one is due to the fact that none of the orientifold projections respecting chiral extended 6D supersymmetry allows to turn on fluxes. This is, on the other hand, consistent with the statement that $(2, 0)$ supergravities do not admit any consistent embedding tensor deformations. Conversely in the non-chiral $(1, 1)$ case, we find a wide range of flux compactifications.

In [52], an analysis of this sort was already presented and all the cases consistent with 6D Lorentz symmetry and $(1, 1)$ supersymmetry were discussed. In each single setup the dictionary was obtained between 10D (11D) fields & fluxes on the one side, and 6D fields & deformations on the other side. The approach used mimics that of [53, 54] designed for orientifold reductions down to 4D. Focusing on compactifications over 4D twisted tori, a vacua scan performed with the aid of the 6D gauged supergravity description showed the existence of a wide landscape of Minkowski (Mkw) vacua, but no maximally symmetric backgrounds with non-vanishing cosmological constant appeared.

In this context, the aim of this thesis is to extend the analysis carried out in [52] to include open string effects such as dynamical brane position moduli and Wilson lines wrapped in internal space, as well as non-Abelian brane gauge groups and non-trivial associated Yang-Mills (YM) flux. While these ingredients are difficult to take into account from a top-down perspective, we show that these are straightforwardly handled from a bottom-up viewpoint, just at the price of including extra vector multiplets within the associated gauged supergravity description. The reason for this is that Lagrangians of half-maximal gauged supergravities (see *e.g.* [55] for the 4D & 5D cases) are fully determined for a given choice of embedding tensor

[1]Some related works in 4D exist for $\mathcal{N} = 4$ [47, 48, 49], $\mathcal{N} = 2$ [50], and more recently, [51].

[56], simply out of imposing consistency and supersymmetry.

At a technical level, the work done in Chapter 6 proves the equivalence between the effective scalar potential obtained from type IIB orientifold reductions and that of a suitable gauged supergravity, with the appropriate amount of vector multiplets accounting for both closed and open string excitations. The highly non-trivial result is a full matching between the scalar potential obtained from gauged supergravity and the one arising from reduction of the bulk action plus the contributions coming from the effective actions of the spacetime filling sources. This matching works even in presence of open string effects such as non-Abelian brane gauge groups and non-vanishing YM internal flux. It is worth remarking that such competing effects between closed and open string sectors in some sense require working at *finite* α'. It still remains to be understood whether this set of α' effects is also physically reliable, besides being mathematically consistent.

Structure of the thesis. This manuscript is organized as follows.

Chapter 2 provides a general overview of fundamental aspects of string theory that are relevant for the rest of the thesis. It begins with the formulation of the non-linear σ-model, which describes the propagation of strings in curved backgrounds, and discusses the emergence of effective low-energy actions through the vanishing of β-functions. These conditions naturally lead to supergravity theories as the low-energy limit of string theory. The chapter also reviews string dualities, focusing on T-duality and S-duality, which play a central role in connecting different regimes of the theory and unifying seemingly distinct descriptions of string backgrounds.

Chapter 3 turns to the study of charged objects in string theory, most notably D-branes and orientifold planes, which are indispensable ingredients in modern developments of the field. The effective actions describing D-branes and O-planes are introduced, and their interplay in Op/Dp systems is discussed. The chapter then presents the corresponding supergravity solutions, including those of Dp-branes, the fundamental string, and the NS5-brane, which serve as classical backgrounds sourced by these objects. Building on this, the AdS/CFT correspondence is introduced, highlighting the role of branes in holography. Finally, the chapter concludes with a discussion of supersymmetry projectors in type IIB theory, which provide a systematic way to characterize the preserved supersymmetry of different brane configurations.

Chapter 4 is devoted to the study of gaugings in supergravity and their higher-dimensional origin. After reviewing the structure of ungauged supergravity theories, with particular attention to their scalar manifolds in lower dimensions, the embedding tensor formalism is introduced as a powerful tool to describe gauged supergravities. The role of fermion shifts and their relation to vacuum solutions is explained, followed by a detailed analysis of the resulting scalar potentials, including the simplifications obtained at the origin of the scalar manifold. The chapter then explores the higher-dimensional origin of these gaugings, covering compactifications on group manifolds, bulk reductions of type II theories, and the appearance of tadpoles and Green–Schwarz mechanisms. This sets the stage for the later analysis of flux compactifications and their relation to gauged supergravity.

Chapter 5 is based on [57]. We discuss D3 – D5 – D7 brane intersections and the form of general solutions in this class. After arguing for the absence of AdS_3 near-horizon geometries in this setup, we briefly discuss the interpretation of these solutions as supersymmetric position-dependent profiles for the YM coupling on the D3 branes due to the presence of D5 – D7 defect branes. Next, we move to D3 – D3 – D7 intersections, where we show how an $\mathrm{AdS}_3 \times S^3 \times \mathbb{T}^2$ geometry with warping over a Riemann surface Σ is obtained by taking the near-horizon limit. Furthermore, we consider a more involved setup, though preserving the same spacetime symmetry as well as the same amount of supersymmetry as the previous cases. The objects intersecting the original stack of D3 branes this time are two differently placed D5s and NS5s, as well as D7 branes. This way, the resulting AdS_3 solutions feature a warping over $\mathbb{T}^3 \times \Sigma$.

Chapter 6 is based on [58]. We firstly review some salient features of Op/Dp systems, the associated light dof's, possible gauge groups and consistency requirements. In Sec. 6.3 we spell out the embedding tensor formulation of 6D $\mathcal{N} = (1, 1)$ gauged supergravities coupled to an arbitrary number of vector multiplets. Then, we analyze the case of IIB reductions including spacetime filling O5/D5 sources and work out the dictionary between the 6D supergravity side and the type IIB side. A parallel analysis is then carried out for O7/D7 sources and later for O9/D9, *i.e.* type I reductions. One of the key results of the paper is the discovery of bulk field strength modifications sourced by the open string vector fields, just like in the heterotic case, where this was due to the Green-Schwarz (GS) mechanism [59]. Indeed, the modifications derived here could be heuristically understood as U-dual versions of GS terms.

Finally, in Chapter 7 we present our conclusions.

Chapter 2

Aspects of String Theory

In this chapter we will review some key aspects of string theory based on [60, 61, 62, 63, 64, 65]. It is structured around three main ideas. First, the non-linear sigma model is introduced through the Polyakov action, which describes string propagation in curved backgrounds and leads, via conformal gauge fixing and boundary conditions, to the equations of motion and the definition of D-branes. Second, by requiring conformal invariance of the action describing a string propagating in a NS-NS background, the vanishing of beta functions imposes some conditions on the background fields, which can be derived from a low-energy effective action. This provides a link between the worldsheet theory and the background dynamics. In this limit, the five consistent superstring theories give rise to five different ten-dimensional supergravity theories. Finally, we review T-duality and S-duality, which not only relate different regimes but also provide insight into how the five superstring theories may be connected within a unified framework.

2.1 | The non-linear σ-model

The relativistic string action can be understood as the extension of the action from relativistic point particles to one-dimensional extended objects. For a point particle, the action is proportional to the proper length of its worldline, which minimizes the path in spacetime. In analogy, the string action is proportional to the proper area of its worldsheet, the two-dimensional surface swept by the string's motion through a D-dimensional space endowed with a metric[1] $g_{\mathcal{MN}}$. This action was proposed by Yoichiro Nambu [66] and Tetsuo Goto [67], and it is called Nambu-Goto action:

$$S_{NG}[x^{\mathcal{M}}(\xi)] = -T \int_{\Sigma} d^2\xi \sqrt{-\det\left[\partial_M X^{\mathcal{M}} \partial_N X^{\mathcal{N}} g_{\mathcal{MN}}(X)\right]}\ , \tag{2.1}$$

where the integral runs over the worldsheet manifold Σ. The coordinates on Σ are $\xi^M = (\tau, \sigma)$, with $M = 0, 1$; here, τ is a time evolution parameter, similar to proper time, while σ parameterizes positions along the string at fixed τ, often taken in the range $[0, \pi]$ or $[0, 2\pi]$ for closed strings. The fields $X^{\mathcal{M}}(\xi)$, with $\mathcal{M} = 0, \ldots, D-1$, are the embedding coordinates that map points on the worldsheet to points in the target spacetime. The negative sign under the square root ensures the expression accounts for the Lorentzian signature, yielding a real value for timelike worldsheets. The constant T represents the string tension, with natural units

[1]Throughout all this work we will adopt the mostly-plus convention, which corresponds to a metric with signature $(D-1, 1)$, where D denotes the dimension of the space-time manifold.

of mass squared. In these units, length is measured as the inverse of mass, allowing T to be interpreted as the mass per unit length of the string. This string tension is related to the Regge slope α' by the expression:

$$T = \frac{1}{2\pi\alpha'} \ , \tag{2.2}$$

and α' also fixes the fundamental scales of string theory: the string mass, m_s, and the fundamental length scale, ℓ_s, of the theory.

$$\ell_s = \frac{1}{m_s} = \sqrt{\alpha'} \ . \tag{2.3}$$

Quantizing the Nambu-Goto action is challenging due to its highly non-linear nature, even in a simple background as Minkowski spacetime, where $g_{\mathcal{MN}} = \eta_{\mathcal{MN}}$, where $\eta_{\mathcal{MN}}$ is the diagonal matrix $(-1, +1, \ldots, +1)$. However, there exists an alternative action that depends quadratically on the derivatives of the worldsheet fields $X^{\mathcal{M}}(\xi)$ and leads to the same classical equations of motion, but to construct this action one needs to introduce an auxiliary worldsheet metric h_{MN}. This is the Polyakov action [68]:

$$S_P[X^{\mathcal{M}}(\xi), h_{MN}(\xi)] = -\frac{T}{2}\int_\Sigma d^2\xi\sqrt{-h}\, h^{MN}\partial_M X^{\mathcal{M}}\partial_N X^{\mathcal{N}} g_{\mathcal{MN}}(X) \ , \tag{2.4}$$

where h is the determinant of h_{MN}. This action is a *non-linear σ-model*, and its variation is given by:

$$\begin{aligned}\delta S_P =& T\int_\Sigma d^2\xi\sqrt{-h}\ \delta X^{\mathcal{M}} g_{\mathcal{MN}}\left[\nabla^2 X^{\mathcal{N}} + h^{MN}\partial_M X^{\mathcal{M}'}\partial_N X^{\mathcal{N}'}\Gamma^{\mathcal{N}}_{\mathcal{M}'\mathcal{N}'}(g)\right] \\ &- \frac{T}{2}\int_\Sigma d^2\xi\sqrt{-h}\ \delta h^{MN} g_{\mathcal{MN}}\left[\partial_M X^{\mathcal{M}}\partial_N X^{\mathcal{N}} - \frac{1}{2}h_{MN}h^{M'N'}\partial_{M'}X^{\mathcal{M}}\partial_{N'}X^{\mathcal{N}}\right] \\ &- T\int_{\partial\Sigma} d\Sigma^M \delta X^{\mathcal{M}}\partial_M X^{\mathcal{N}} g_{\mathcal{MN}},\end{aligned} \tag{2.5}$$

where $d\Sigma^M$ is the boundary measure element, capturing both the infinitesimal boundary length and the outward normal in the M-th direction. Since no kinetic term for the worldsheet metric appears in the action, the variation of the action with respect to the metric imposes that the energy–momentum tensor must vanish. This is known as the Virasoro constraint, and the stress-energy tensor is

$$T_{MN} = \frac{2}{T}\frac{1}{\sqrt{-h}}\frac{\delta S}{\delta h^{MN}} = \partial_M X^{\mathcal{M}}\partial_N X^{\mathcal{N}} g_{\mathcal{MN}} - \frac{1}{2}h_{MN}h^{M'N'}\partial_{M'}X^{\mathcal{M}}\partial_{N'}X^{\mathcal{N}} g_{\mathcal{MN}}. \tag{2.6}$$

This constraint is used to eliminate h_{MN} from the Polyakov action by substituting:

$$h_{MN} = \frac{2}{g_{M'}{}^{M'}} g_{MN}, \tag{2.7}$$

where the induced metric g_{MN} is defined as $g_{MN} = \partial_M X^{\mathcal{M}}\partial_N X^{\mathcal{N}} g_{\mathcal{MN}}$. Notice that the proportionality factor $g_{M'}{}^{M'} = g_{M'N'}h^{M'N'}$ remains undetermined because the energy-momentum tensor is traceless off-shell. Moreover, the Polyakov action admits Weyl symmetry, meaning the worldsheet metric can be locally rescaled:

$$h_{MN} \rightarrow \Omega^2(\xi)\; h_{MN}\;, \tag{2.8}$$

where $\Omega(\xi)$ is an arbitrary, non-vanishing smooth function on the worldsheet.

In two dimensions, one can use reparametrizations together with Weyl symmetry to set $h_{MN} = \eta_{MN}$. This is the so-called *conformal gauge.*

$$S_P = \frac{T}{2}\int_\Sigma d^2\xi\, \eta^{MN}\partial_M X^{\mathcal{M}}\partial_N X^{\mathcal{N}} g_{\mathcal{M}\mathcal{N}}. \tag{2.9}$$

The equations of motion follow by performing the variation of the Polyakov action (in conformal gauge) from τ_i to τ_f:

$$\delta S_P = -T\int_{\tau_i}^{\tau_f} d\tau \int_0^{\sigma_0} d\sigma\; \left[\delta X^{\mathcal{M}}\eta^{MN}\partial_M\partial_N X^{\mathcal{N}} g_{\mathcal{M}\mathcal{N}} + \partial_M\left(\eta^{MN}\delta X^{\mathcal{M}}\partial_N X^{\mathcal{N}} g_{\mathcal{M}\mathcal{N}}\right)\right]. \tag{2.10}$$

Although it is not necessary, it is a sufficient condition requiring that each term vanishes to ensure that the overall variation δS_P is zero. If the first term vanishes, this gives us the two-dimensional wave equation:

$$\partial_M\partial^M X^{\mathcal{M}} = 0. \tag{2.11}$$

Any solution of the two-dimensional wave equation can be written as the sum of independent left and right-moving modes, $X_+^{\mathcal{M}}(\sigma^+)$ and $X_-^{\mathcal{M}}(\sigma^-)$ respectively:

$$X^{\mathcal{M}}(\tau,\sigma) = X_-^{\mathcal{M}}(\sigma_-) + X_+^{\mathcal{M}}(\sigma_+), \tag{2.12}$$

where

$$\sigma^+ = \tau + \sigma, \qquad \sigma^- = \tau - \sigma. \tag{2.13}$$

To determine these functions, boundary conditions must be applied at the endpoints of the string. The second term in (2.10) is a boundary term, that can rewritten as

$$-\left[\int_0^{\pi} d\sigma\, X^{\mathcal{M}}\partial_\tau \delta X_{\mathcal{M}}\right]_{\tau_i}^{\tau_f} + \left[\int_{\tau_i}^{\tau_f} d\tau\, \delta X^{\mathcal{M}}\partial_\sigma X_{\mathcal{M}}\right]_0^{\sigma_0} = \left[\int_{\tau_i}^{\tau_f} d\tau\, \delta X^{\mathcal{M}}\partial_\sigma X_{\mathcal{M}}\right]_0^{\sigma_0}, \tag{2.14}$$

where the first term in the left hand side vanishes because the endpoints are fixed at t_i and t_f. We want the remaining term to vanish, to satisfy that we can distinguish two topologically distinct cases, open strings and closed strings.

In the closed string case, the string forms a loop without endpoints and we adopt the convention that the integration limit $\sigma_0 = 2\pi$. Since the string is closed, it must satisfy the following periodic conditions:

$$X^{\mathcal{M}}(\tau,0) = X^{\mathcal{M}}(\tau,2\pi), \qquad \partial_\sigma X^{\mathcal{M}}(\tau,0) = \partial_\sigma X^{\mathcal{M}}(\tau,2\pi). \tag{2.15}$$

These conditions ensure that the boundary term vanishes, and therefore implies that $\delta S_P = 0$ on-shell.

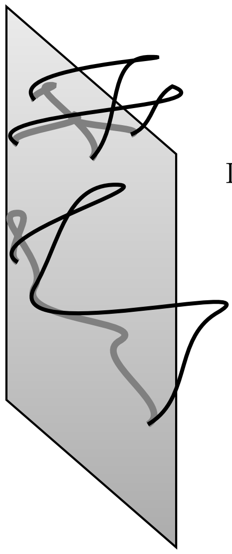

Figure 2.1: *Open strings attached to a Dp-brane. Dirichlet boundary conditions fix the endpoints of the string onto the brane.*

On the other hand, for the open string case, we adopt the convention $\sigma_0 = \pi$. We will impose the following condition at $\sigma = 0$ and $\sigma = \pi$:

$$\delta X^{\mathcal{M}} \partial_\sigma X_{\mathcal{M}} = 0. \tag{2.16}$$

Therefore $\partial_\sigma X_{\mathcal{M}} = 0$ or $\delta X^{\mathcal{M}} = 0$ at those points. The behavior of open strings, especially at their endpoints, is determined by boundary conditions. These conditions can be classified into two types: Neumann and Dirichlet boundary conditions.

The Neumann (N) boundary condition imposes that the variation of the string's position along the direction normal to the endpoints of the string vanishes:

$$\partial_\sigma X_{\mathcal{M}} = 0, \tag{2.17}$$

at $\sigma = 0$ and $\sigma = \pi$. So, as $\delta X^{\mathcal{M}}$ is free, the endpoints can move with no restrictions.

On the other hand, the Dirichlet (D) boundary condition fixes the string's endpoints position:

$$\delta X^{\mathcal{M}} = 0, \tag{2.18}$$

Let us discuss the case in more detail. The physical objects that constrain these open string endpoints are characterized by their dimensionality. These objects are known as D-branes, where the "D" is short for Dirichlet. A Dp-brane is defined as an object that has p spatial dimensions. Thus, for open strings, we can rewrite the previous boundary conditions as:

$$\partial_+ X_+^{\mathcal{M}} = -\partial_- X_-^{\mathcal{M}} \quad \text{(Neumann condition)}, \tag{2.19}$$

$$\partial_+ X_+^{\mathcal{M}} = \partial_- X_-^{\mathcal{M}} \quad \text{(Dirichlet condition)}, \tag{2.20}$$

where $\partial_\pm \equiv \partial_{\sigma_\pm} = \partial_\tau \pm \partial_\sigma$.

Beyond the classical classification of open vs. closed and oriented vs. unoriented -which fixes the admissible worldsheet topologies- it is precisely the boundary conditions and orientation that, after quantization, determine the physical content of the theory. The procedures of quantization (canonical or path-integral/BRST), the modal expansion of the string, the Virasoro constraints and the various projection conditions (for example, GSO projections and Chan-Paton factors) convert classical oscillation modes into quantum excitations and thereby select the spectrum of states. Heuristically, different choices yield different spectra (open oriented sectors can produce gauge vector states; closed oriented sectors include the graviton, dilaton and antisymmetric tensor; certain formulations admit tachyonic modes that signal instabilities), and consistency requirements such as anomaly cancellation and critical dimension further restrict which spectra are physically acceptable.

Although these quantization and spectrum issues are important background for understanding the implications of the geometric distinctions just described, they will not be developed further in this thesis.

2.2 | Low-energy limit string effective actions

Starting from the Polyakov action, we can include additional background fields such as the Kalb-Ramond 2-form $B_{\mathcal{M}\mathcal{N}}$, and the dilaton Φ. The action describing a string propagating in a background with the massless fields $g_{\mathcal{M}\mathcal{N}}(X)$, $B_{\mathcal{M}\mathcal{N}}(X)$, and $\Phi(X)$ is given by [62]:

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\,\sqrt{-h}\,\Big(h^{MN}\,\partial_M X^{\mathcal{M}}\partial_N X^{\mathcal{N}}\,g_{\mathcal{M}\mathcal{N}}(X) + \epsilon^{MN}\partial_M X^{\mathcal{M}}\partial_N X^{\mathcal{N}} B_{\mathcal{M}\mathcal{N}}(X) + \alpha'\mathcal{R}_{(h)}\Phi(X)\Big)\,, \tag{2.21}$$

where $\mathcal{R}_{(h)}$ denotes the Ricci scalar associated with the worldsheet induced metric h.

2.2.1. Beta functions

In quantum field theory, the dependence of the coupling constants of a theory with respect to the energy scale is governed by the so-called β-function. Therefore, a vanishing β-function indicates scale-invariant behavior, meaning that the theory remains unchanged for everyenergy scale.

In order to understand whether conformal invariance is preserved at a quantum level, one considers the trace of the stress-energy tensor, $T^M{}_M$. This trace may receive contributions from three different types fields, each giving rise to a corresponding beta function [69, 62].

$$T^M{}_M = -\frac{1}{2\alpha'}\beta^g_{\mathcal{M}\mathcal{N}}h^{MN}\partial_M X^{\mathcal{M}}\partial_N X^{\mathcal{N}} - \frac{1}{2\alpha'}\beta^B_{\mathcal{M}\mathcal{N}}\varepsilon^{MN}\partial_M X^{\mathcal{M}}\partial_N X^{\mathcal{N}} - \frac{1}{2}\beta^\Phi R_{(h)}, \tag{2.22}$$

where $\beta^g_{\mathcal{M}\mathcal{N}}$, $\beta^B_{\mathcal{M}\mathcal{N}}$, and β^Φ are the beta functions associated with the metric $g_{\mathcal{M}\mathcal{N}}$, the antisymmetric tensor $B_{\mathcal{M}\mathcal{N}}$, and the dilaton Φ, respectively.

Although we will not enter into the calculations of the one-loop beta functions here, the results can be summarized as follows [70, 69, 62]:

$$\beta^g_{\mathcal{M}\mathcal{N}} = -\alpha' \left(R_{\mathcal{M}\mathcal{N}} + 2\nabla_{\mathcal{M}}\nabla_{\mathcal{N}}\Phi - \frac{1}{4} H_{\mathcal{M}\mathcal{M}'\mathcal{N}'} H_{\mathcal{N}}{}^{\mathcal{M}'\mathcal{N}'} \right) + \mathcal{O}(\alpha'^2), \tag{2.23}$$

$$\beta^B_{\mathcal{M}\mathcal{N}} = \alpha' \left(-\frac{1}{2} \nabla^{\mathcal{M}'} H_{\mathcal{M}'\mathcal{M}\mathcal{N}} + \nabla^{\mathcal{M}'}\Phi\, H_{\mathcal{M}'\mathcal{M}\mathcal{N}} \right) + \mathcal{O}(\alpha'^2), \tag{2.24}$$

$$\beta^\Phi = \frac{D-26}{6} + \alpha' \left(\nabla_{\mathcal{M}}\Phi\, \nabla^{\mathcal{M}}\Phi - \frac{1}{2}\nabla^2\Phi - \frac{1}{24} H_{\mathcal{M}\mathcal{N}\mathcal{M}'} H^{\mathcal{M}\mathcal{N}\mathcal{M}'} \right) + \mathcal{O}(\alpha'^2), \tag{2.25}$$

where the field strenght H in components is

$$H_{\mathcal{M}\mathcal{N}\mathcal{P}} \equiv 3\, \partial_{[\mathcal{M}} B_{\mathcal{N}\mathcal{P}]}. \tag{2.26}$$

Conformal invariance holds when these beta functions vanish, *i.e.*,

$$\beta^g_{\mathcal{M}\mathcal{N}} = \beta^B_{\mathcal{M}\mathcal{N}} = \beta^\Phi = 0. \tag{2.27}$$

The conditions $\beta^g_{\mathcal{M}\mathcal{N}} = \beta^B_{\mathcal{M}\mathcal{N}} = \beta^\Phi = 0$ can be interpreted as the equations of motion governing the background fields in which the string propagates. We therefore turn our attention to the low-energy description that derives these relations from an action principle. This effective action is given (up to α') by [70, 69, 62]

$$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-g}\, e^{-2\Phi} \left[\mathcal{R} + 4\nabla_{\mathcal{M}}\Phi\nabla^{\mathcal{M}}\Phi - \frac{1}{12}|H_{(3)}|^2 - \frac{2(D-26)}{3\alpha'} \right], \tag{2.28}$$

where $|H_{(3)}|^2 = H_{\mathcal{M}\mathcal{N}\mathcal{P}} H^{\mathcal{M}\mathcal{N}\mathcal{P}}$, g is the determinant of the target spacetime metric $g_{\mathcal{M}\mathcal{N}}$, $\mathcal{R}$ is the Ricci scalar, and ∇ is the covariant derivative associated with $g_{\mathcal{M}\mathcal{N}}$. In D spacetime dimensions, the gravitational coupling parameter κ_D takes the form

$$\kappa_D = \frac{1}{\sqrt{4\pi}} g_s (2\pi l_s)^{\frac{D-2}{2}}, \tag{2.29}$$

with $g_s = \lim_{X\to\infty} \Phi(X)$. This constant κ_D is related to the D-dimensional Newton constant G_D as follows:

$$\kappa_D^2 = 8\,\pi\, G_D. \tag{2.30}$$

Note that the metric in the action (2.28) is the same as the one appearing in the non-linear σ model. For this reason, it is known as the string-frame metric. We can also perform the conformal rescaling:

$$g_{\mathcal{M}\mathcal{N}} = e^{\frac{4\Phi}{D-2}} g^E_{\mathcal{M}\mathcal{N}}. \tag{2.31}$$

By rescaling the metric this way, the dilaton no longer multiplies the Ricci scalar, yielding a standard Einstein–Hilbert term. That is why $g^E_{\mathcal{M}\mathcal{N}}$ is called Einstein-frame metric.

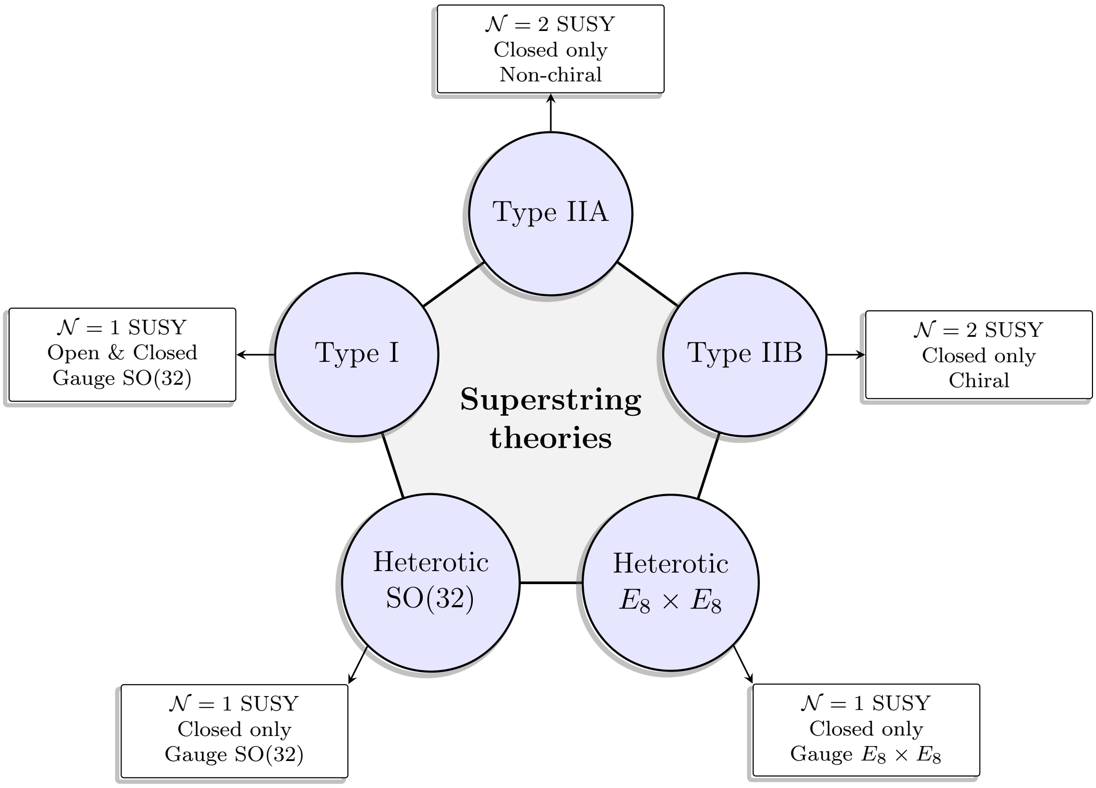

Figure 2.2: *The five consistent superstring theories in 10D.*

2.2.2. Supergravities

Bosonic string theory predicts the existence of tachyons, which are states associated with a negative mass squared, implying an instability in the vacuum [71]. Additionally, as the bosonic model does not incorporate fermionic fields, it is unable to describe particles like quarks and leptons that are fermionic degrees of freedom. For these reasons, bosonic string theory alone cannot serve as a fully realistic framework for Nature.

Fermionic degrees of freedom can be added through the introduction of supersymmetry, which not only extends the bosonic framework but also eliminates the presence of tachyons, leading to what is known as superstring theory. The requirement of anomaly cancellation and consistency conditions in ten dimensions leads to exactly five consistent superstring theories: type I, type IIA, type IIB, and two heterotic string theories based on the gauge groups SO(32) and $E_8 \times E_8$ [59, 72, 73]. These theories differ in several key aspects: the type of supersymmetry they realize (type I and heterotic strings have $\mathcal{N} = 1$ supersymmetry, while type II theories have $\mathcal{N} = 2$), the nature of the strings involved (open and closed in type I, only closed in the rest), and the structure of their gauge symmetries (present only in type I and heterotic strings). Moreover, type IIA and type IIB differ in the chirality of their fermionic sectors: type IIA is non-chiral, while type IIB is chiral. These characteristics are summarized in Figure 2.2.

Each of these theories includes the fields $g_{\mathcal{MN}}$, Φ, and, in all cases except type I, the antisymmetric two-form $B_{\mathcal{MN}}$, and therefore are known as the common sector. In addition, each theory contains a distinct set of Ramond–Ramond (RR) or gauge fields that distinguish their low-energy spectra.

For each theory, the low-energy effective action[2] that governs the dynamics of these fields in $D = 10$ dimensional spacetime decomposes as:

$$S = S_{\text{bos}} + S_{\text{fermi}} \ , \tag{2.32}$$

where S_{fermi} accounts for the interactions of the spacetime fermions, which we will not describe here. However, we will briefly outline the low-energy bosonic action S_{bos} for each of the five supergravity theories [74, 65, 75].

- **Type IIA**: The additional bosonic fields in type IIA include a 1-form $C_{(1)}$ and a 3-form $C_{(3)}$. The dynamics of these fields are governed by the Ramond-Ramond sector of the action, which is written as

$$\begin{aligned} S_{\text{bos}} = \frac{1}{2\kappa_{10}^2}\Bigg[\int d^{10}x\sqrt{-g}\Bigg(e^{-2\Phi}\left(\mathcal{R} + 4\partial_{\mathcal{M}}\Phi\,\partial^{\mathcal{M}}\Phi - \frac{|H_{(3)}|^2}{2\cdot 3!}\right) - \frac{1}{2}\left(\frac{|F_{(2)}|^2}{2!} + \frac{|F_{(4)}|^2}{4!}\right)\Bigg) \\ - \frac{1}{2}\int dC_{(3)} \wedge dC_{(3)} \wedge B_{(2)}\Bigg], \end{aligned} \tag{2.33}$$

where the field strengths are given by:

$$F_{(2)} = dC_{(1)}, \tag{2.34}$$
$$F_{(4)} = dC_{(3)} - H_{(3)} \wedge C_{(1)}, \tag{2.35}$$

where d denotes the exterior derivative. Note that the last term in the action is independent of the metric, and therefore is a topological term, that is known as the Chern-Simons term.

It is possible to introduce a deformation by a mass parameter m into type IIA supergravity, which is called the Romans mass. This theory is known as massive type IIA supergravity. To incorporate the mass parameter, the definitions of the field strengths are modified as follows:

$$F_{(2)} = dC_{(1)} + mB_{(2)}, \tag{2.36}$$
$$F_{(4)} = dC_{(3)} - H_{(3)} \wedge C_{(1)} + \frac{m}{2}B_{(2)} \wedge B_{(2)}, \tag{2.37}$$

and the bosonic action is

$$\begin{aligned} S_{\text{bos}} = \frac{1}{2\kappa_{10}^2}\Bigg[\int d^{10}x\sqrt{-g}\left(e^{-2\Phi}\left(\mathcal{R} + 4\partial_{\mathcal{M}}\Phi\,\partial^{\mathcal{M}}\Phi - \frac{|H_{(3)}|^2}{2\cdot 3!}\right) - \frac{1}{2}\left(m^2 + \frac{|F_{(2)}|^2}{2!} + \frac{|F_{(4)}|^2}{4!}\right)\right) \\ - \frac{1}{2}\int\left(dC_{(3)} \wedge dC_{(3)} \wedge B_{(2)} + \frac{m}{3}dC_{(3)} \wedge B_{(2)}^3 + \frac{m^2}{20}B_{(2)}^5\right)\Bigg], \end{aligned} \tag{2.38}$$

where $B_{(2)}^n$ is the wedge product of n $B_{(2)}$ forms.

[2]Each of these low-energy effective actions corresponds to a supergravity theory that shares its name with the associated superstring theory. The concept of supergravity will be reviewed in Chapter 4.

Democratic formulation

Massive type IIA supergravity [76] admits a democratic formulation [77]. The bosonic sector thereof contains the usual NS-NS fields $\{g, B_{(2)}, \Phi\}$, whereas the R-R degrees of freedom are doubled $\{C_{2p+1}\}_{p=0,1,2,3,4}$. The following pseudoaction can be used to obtain the equations of motion in the boson part:

$$S_{\rm bos} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\,\sqrt{-g}\left(e^{-2\Phi}\left(\mathcal{R} + 4\partial_{\mathcal{M}}\Phi\,\partial^{\mathcal{M}}\Phi - \frac{|H_{(3)}|^2}{2\cdot 3!}\right) - \frac{1}{4}\sum_{p=0}^{5}\frac{|F_{(2p)}|^2}{(2p)!}\right)\,. \tag{2.39}$$

It is worth mentioning that this expression is called a pseudoaction because it must be supplemented by the following *duality relations*:

$$F_{(0)} = \star F_{(10)}\;, \qquad F_{(2)} = -\star F_{(8)}\;, \qquad F_{(4)} = \star F_{(6)}\;. \tag{2.40}$$

This way, the original number of degrees of freedom in the R-R sector is restored.

The field strengths appearing in the pseudoaction (2.39) are given by

$$H_{(3)} = dB_{(2)}\,, \quad \mathbf{F} = d\mathbf{C} + m\,e^{B_{(2)}} - H_{(3)}\wedge\mathbf{C}, \tag{2.41}$$

where

$$\mathbf{F} = \sum_n F_{(2n)}, \qquad \mathbf{C} = \sum_n C_{(2n-1)}. \tag{2.42}$$

Making the formal sums in (2.41) explicit, we can write the field strengths of the R-R fields as

$$\begin{aligned}
F_{(0)} &= m \\
F_{(2)} &= dC_{(1)} + F_{(0)}B_{(2)}, \\
F_{(4)} &= dC_{(3)} - H_{(3)}\wedge C_{(1)} + F_{(0)}\frac{B_{(2)}^2}{2!}, \\
F_{(6)} &= dC_{(5)} - H_{(3)}\wedge C_{(3)} + F_{(0)}\frac{B_{(2)}^3}{3!}, \\
F_{(8)} &= dC_{(7)} - H_{(3)}\wedge C_{(5)} + F_{(0)}\frac{B_{(2)}^4}{4!}, \\
F_{(10)} &= dC_{(9)} - H_{(3)}\wedge C_{(7)} + F_{(0)}\frac{B_{(2)}^5}{5!}.
\end{aligned} \tag{2.43}$$

In particular, Roman's mass contribution is encoded in (2.43), with constant m.

Because of their definitions, the field strengths naturally satisfy the modified Bianchi identities

$$dH_{(3)} = 0 \quad , \qquad dF_{(0)} = 0 \quad , \qquad dF_{(2)} + H_{(3)}\wedge F_{(0)} = 0 \quad ,$$
$$dF_{(4)} + H_{(3)}\wedge F_{(2)} = 0 \quad , \qquad dF_{(6)} + H_{(3)}\wedge F_{(4)} = 0 \quad ,$$

$$dF_{(8)} + H_{(3)} \wedge F_{(6)} = 0 \quad , \qquad dF_{(10)} = 0 \quad .$$

Moreover, (2.40) maps the modified Bianchi identities to the equations of motion for the form fields in the NS-NS and the R-R sector:

$$\begin{aligned} d \star F_{(10)} &= 0 \quad , \qquad d \star F_{(8)} + H_{(3)} \wedge \star F_{(10)} = 0 \quad , \\ d \star F_{(6)} + H_{(3)} \wedge \star F_{(8)} &= 0 \quad , \qquad d \star F_{(4)} + H_{(3)} \wedge \star F_{(6)} = 0 \quad , \\ d \star F_{(2)} + H_{(3)} \wedge \star F_{(4)} &= 0 \quad , \qquad d \star F_{(0)} = 0 \quad , \\ d\big(e^{-2\Phi} \star H_{(3)}\big) - \frac{1}{2} \sum_{p=0}^{4} F_{(2p)} \wedge \star F_{(2p+2)} &= 0 \quad . \end{aligned}$$

The R-R sector can therefore be written as

$$d \star F_{(2p)} + H_{(3)} \wedge \star F_{(2p+2)} = 0 \,. \tag{2.44}$$

The equations of motions for the dilaton, instead, read

$$\nabla_{\mathcal{M}} \nabla^{\mathcal{M}} \Phi - \nabla_{\mathcal{M}} \Phi \nabla^{\mathcal{M}} \Phi + \frac{1}{4} \mathcal{R} - \frac{1}{8 \cdot 3!} |H_{(3)}|^2 = 0 \quad , \tag{2.45}$$

whereas those for the metric are

$$\begin{aligned} 0 = e^{-2\Phi} \big(\mathcal{R}_{\mathcal{M}\mathcal{N}} + 2\nabla_{\mathcal{M}} \nabla_{\mathcal{N}} \Phi - \frac{1}{4} H_{\mathcal{M}\mathcal{M}'\mathcal{N}'} H_{\mathcal{N}}{}^{\mathcal{M}'\mathcal{N}'}\big) - \frac{1}{2} (F^2_{(2)})_{\mathcal{M}\mathcal{N}} - \frac{1}{2 \cdot 3!} (F^2_{(4)})_{\mathcal{M}\mathcal{N}} + \\ + \frac{1}{4} g_{\mathcal{M}\mathcal{N}} \big(|F_{(0)}|^2 + \frac{1}{2!} |F_{(2)}|^2 + \frac{1}{4!} |F_{(4)}|^2\big) \quad . \end{aligned} \tag{2.46}$$

- **Type IIB**: The additional bosonic fields include a scalar $C_{(0)}$, a 2-form $C_{(2)}$, and a 4-form $C_{(4)}$. The action is expressed as

$$\begin{aligned} S_{\text{bos}} = \frac{1}{2\kappa_{10}^2} \Bigg[\int d^{10}x \sqrt{-g} \, \Bigg(e^{-2\Phi} \Bigg(\mathcal{R} + 4 \partial_{\mathcal{M}} \Phi \, \partial^{\mathcal{M}} \Phi - \frac{|H_{(3)}|^2}{2 \cdot 3!} \Bigg) \\ - \frac{1}{2} \Bigg(|F_{(1)}|^2 - \frac{|F_{(3)}|^2}{3!} - \frac{|F_{(5)}|^2}{5!} \Bigg) \Bigg) - \frac{1}{2} \int C_{(4)} \wedge H_{(3)} \wedge dC_{(2)} \Bigg], \end{aligned} \tag{2.47}$$

where the field strength are

$$F_{(1)} = dC_{(0)}, \tag{2.48}$$

$$F_{(3)} = dC_{(2)} - C_{(0)} H_{(3)}, \tag{2.49}$$

$$F_{(5)} = dC_{(4)} - \frac{1}{2} C_{(2)} \wedge H_{(3)} + \frac{1}{2} B_{(2)} \wedge dC_{(2)}. \tag{2.50}$$

An additional condition in type IIB theory, which is not directly derivable from a Lagrangian, requires this field to be self-dual:

$$F_{(5)} = \star F_{(5)} . \tag{2.51}$$

Democratic formulation

We can rewrite type IIB supergravity in its democratic formulation [77] is described in terms of the common NS-NS sector $\{g, B_{(2)}, \Phi\}$ coupled to even form fields $\{C_{(2p)}\}_{p=0,1,2,3,4}$. [3] The (bosonic) dynamics of the theory can be derived from the following *pseudoaction*:

$$S_{\rm bos} = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-g} \left(e^{-2\Phi} \left(\mathcal{R} + 4\partial_{\mathcal{M}}\Phi\, \partial^{\mathcal{M}}\Phi - \frac{|H_{(3)}|^2}{2\cdot 3!} \right) - \frac{1}{4}\sum_{p=0}^{4} \frac{|F_{(2p+1)}|^2}{(2p+1)!} \right) . \tag{2.52}$$

As in the previous case of the type IIA, this expression is called a pseudoaction because it must be supplemented by the following *duality relations*

$$F_{(9)} \overset{!}{=} \star F_{(1)} \ , \quad F_{(7)} \overset{!}{=} - \star F_{(3)} \ , \quad F_{(5)} \overset{!}{=} \star F_{(5)} \ . \tag{2.53}$$

that yield the correct number of propagating degrees of freedom and hence allow for an on-shell realization of supersymmetry.

The field strengths read

$$\begin{aligned} F_{(1)} &= dC_{(0)} \ , & F_{(3)} &= dC_{(2)} - H_{(3)} \wedge C_{(0)} \ , \\ F_{(5)} &= dC_{(4)} - H_{(3)} \wedge C_{(2)} \ , & F_{(7)} &= dC_{(6)} - H_{(3)} \wedge C_{(4)} \ , \\ F_{(9)} &= dC_{(8)} - H_{(3)} \wedge C_{(6)} \ , & H_{(3)} &= dB_{(2)} \ , \end{aligned}$$

which are designed to automatically satisfy the following (modified) Bianchi identities

$$\begin{aligned} dH_{(3)} = 0 \ , \quad dF_{(1)} &= 0 \ , & dF_{(3)} - H_{(3)} \wedge F_{(1)} &= 0 \ , \\ dF_{(5)} - H_{(3)} \wedge F_{(3)} &= 0 \ , & dF_{(7)} - H_{(3)} \wedge F_{(5)} &= 0 \ , \\ dF_{(9)} - H_{(3)} \wedge F_{(7)} &= 0 \ . \end{aligned} \tag{2.54}$$

By varying (2.52), one obtains the following set of equations of motion

$$\Box\Phi \ - \ (\partial\Phi)^2 \ + \ \frac{1}{4}\,\mathcal{R} \ - \ \frac{1}{8\cdot 3!}\,|H_{(3)}|^2 \ = \ 0 \ , \tag{2.55}$$

for the 10D dilaton Φ,

$$d(\star F_{(1)}) + H_{(3)} \wedge (\star F_{(3)}) = 0 \ , \qquad d(\star F_{(3)}) + H_{(3)} \wedge F_{(5)} = 0 \ , \tag{2.56}$$

$$d(\star F_{(5)}) - H_{(3)} \wedge F_{(3)} = 0 \ , \qquad d(e^{-2\Phi} \star H_{(3)}) + \frac{1}{2}\sum_p \star F_{(p)} \wedge F_{(p-2)} = 0 \ , \tag{2.57}$$

[3] See [125] for a type IIB formulation where the $SL(2,\mathbb{R})$ 2-form doublet and the scalar fields are democratized.

for the form fields, and finally the (trace reversed) Einstein equations

$$0 = e^{-2\Phi}\left(\mathcal{R}_{\mathcal{M}\mathcal{N}} + 2\nabla_{\mathcal{M}}\nabla_{\mathcal{N}}\Phi - \frac{1}{4}H_{\mathcal{M}\mathcal{M}'\mathcal{N}'}H_{\mathcal{N}}{}^{\mathcal{M}'\mathcal{N}'}\right) - \frac{1}{2}(F^2_{(1)})_{\mathcal{M}\mathcal{N}}$$
$$- \frac{1}{2\cdot 2!}(F^2_{(3)})_{\mathcal{M}\mathcal{N}} - \frac{1}{4\cdot 4!}(F^2_{(5)})_{\mathcal{M}\mathcal{N}} + \frac{1}{4}g_{\mathcal{M}\mathcal{N}}\left(|F_{(1)}|^2 + \frac{1}{3!}|F_{(3)}|^2\right) . \tag{2.58}$$

- **Type I**: This theory arises when considering Type IIB in the presence of D9/O9. While the presence of the 32 D9 branes implies the existence of a set of non-Abelian gauge one-form potentials $\mathcal{A}^I_{(1)}$, transforming in the vector representation of the SO(32) gauge group, the RR two-form potential $C_{(2)}$ survives the projection of the O9. At low energies, the effective ten-dimensional action can be written as

$$S_{\text{bos}} = \frac{1}{2\,\kappa_{10}^2}\int d^{10}x\,\sqrt{-g}\,\left(e^{-2\Phi}\left(\mathcal{R} + 4\partial_{\mathcal{M}}\Phi\,\partial^{\mathcal{M}}\Phi - \frac{|\tilde{F}_{(3)}|^2}{2\cdot 3!}\right) - e^{-\Phi}\,\frac{\text{Tr}_v(|\mathcal{F}_{(2)}|^2)}{2\cdot 2!}\right), \tag{2.59}$$

where Tr_v is the gauge trace in the vector representation, and the field strengths are defined as

$$\mathcal{F}_{(2)} = d\mathcal{A}_{(1)} + i\mathcal{A}_{(1)}\wedge\mathcal{A}_{(1)} \quad \text{and} \quad \tilde{F}_{(3)} = dC_{(2)} - \frac{\alpha'}{4}\omega_{(3)}, \tag{2.60}$$

where $\omega_{(3)}$ is the Chern-Simons 3-form given by:

$$\omega_{(3)} = \mathcal{A}_{(1)}\wedge d\mathcal{A}_{(1)} - \frac{2i}{3}\text{Tr}_v(\mathcal{A}_{(1)}\wedge\mathcal{A}_{(1)}\wedge\mathcal{A}_{(1)}), \tag{2.61}$$

with $\mathcal{A}_{(1)} = \mathcal{A}^I_{(1)}t_I$, where t_I are the generators of the SO(32) gauge group.

- **Heterotic Strings:** Both variants of the heterotic theory are given by the introduction of non-Abelian gauge one-form potentials, $\mathcal{A}^I_{(1)}$, in the vector representation of the gauge group SO(32) or $E_8\times E_8$, respectively. The action S_2 in this case is:

$$S_{\text{bos}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-g}\,e^{-2\Phi}\,\left(\mathcal{R} + 4\partial_{\mathcal{M}}\Phi\,\partial^{\mathcal{M}}\Phi - \frac{1}{2\cdot 3!}|\tilde{H}_{(3)}|^2 - \frac{\alpha'}{2\cdot 2!}\text{Tr}_v\,(|\mathcal{F}_{(2)}|^2)\right). \tag{2.62}$$

The term $\tilde{H}_{(3)}$ appearing in the action is defined as $\tilde{H}_{(3)} = dB_{(2)} - \alpha'(\omega_{(3)} - \omega^{\text{grav}}_{(3)})/4$, where ω_3 and $\mathcal{F}_{(2)}$ are the same as in the type I, and $\omega^{\text{grav}}_{(3)}$ is defined as the analogous to equation (2.61) but involving the spin connection instead, with $d\omega^{\text{grav}}_{(3)} = \text{Tr}\,\mathcal{R}^2_{(2)}$, where $\mathcal{R}_{(2)}$ is the curvature 2-form.

2.3 | Dualities

Dualities in string theory establish equivalences between different formulations or regimes of the theory or between different theories. In particular, they reveal that what initially appears as distinct descriptions of physical phenomena can be reinterpreted as different perspectives on the same underlying framework.

In this section we will study T- and S-duality. T-duality relates string theories compactified on circles of different radii, showing that strings perceive large and small dimensions equivalently

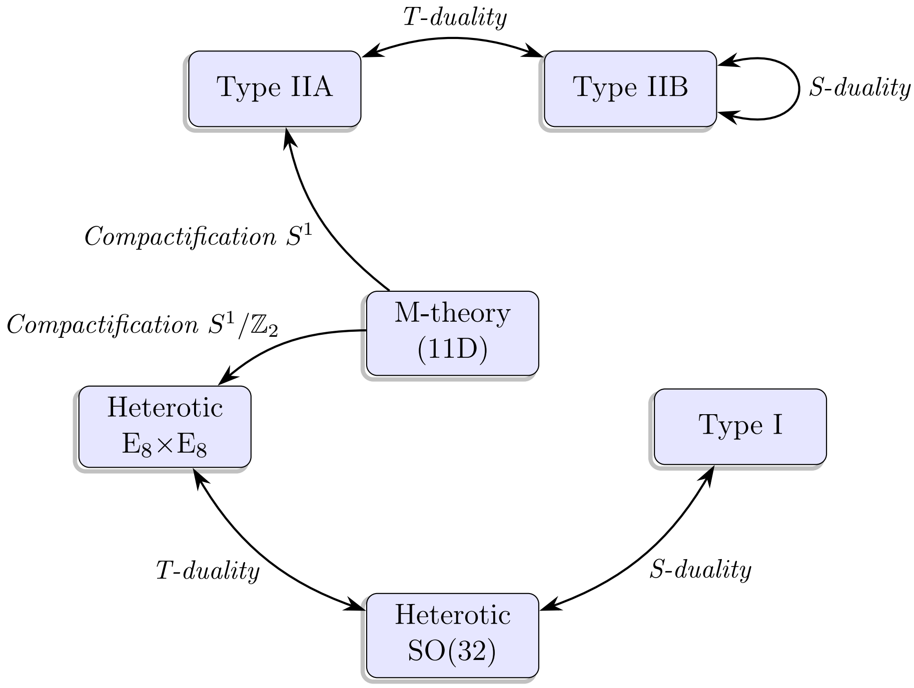

Figure 2.3: *Web of dualities.*

when momentum and winding number[4] are exchanged. S-duality connects theories at strong and weak coupling, allowing for a unified treatment of interactions across different energy scales. Another relevant duality that we will not discuss here is U-duality, that can be constructed from T- and S-duality.

Hull and Townsend [78], along with Witten [79], proposed that these theories can be viewed as different limiting cases of a more fundamental framework in eleven dimensions, known as M-theory. The web of dualities relating the aforementioned theories is shown in figure 2.3 [80].

2.3.1. T-duality

We begin by reviewing T-duality in the context of closed bosonic strings propagating in a spacetime with one compactified spatial dimension.

The solution of the Polyakov action in the conformal gauge is, as we saw previously in (2.12):

$$X^{\mathcal{M}}(\tau,\sigma) = X^{\mathcal{M}}_-(\sigma_-) + X^{\mathcal{M}}_+(\sigma_+), \tag{2.63}$$

where the Fourier expansion is

$$X^{\mathcal{M}}_-(\sigma_-) = \frac{x^{\mathcal{M}}_-}{2} + \alpha' p^{\mathcal{M}}_- \sigma_- + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{\alpha^{\mathcal{M}}}{n} e^{-in\sigma_-}, \tag{2.64}$$

$$X^{\mathcal{M}}_+(\sigma_+) = \frac{x^{\mathcal{M}}_+}{2} + \alpha' p^{\mathcal{M}}_+ \sigma_+ + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{\tilde{\alpha}^{\mathcal{M}}}{n} e^{-in\sigma_+}. \tag{2.65}$$

[4]The winding number is the number of times the string wraps around the circular dimension.

As $X^{\mathcal{M}}$ is real, we need $\alpha_{-n}^{\mathcal{M}} = (\alpha_n^{\mathcal{M}})^*$ and $\tilde{\alpha}_{-n}^{\mathcal{M}} = (\tilde{\alpha}_n^{\mathcal{M}})^*$, because in this case, for each n:

$$\frac{i}{n}\alpha_n^{\mathcal{M}} e^{-in\sigma_-} + \frac{i}{-n}\alpha_{-n}^{\mathcal{M}} e^{-i(-n)\sigma_-} = 2\,\mathrm{Re}\left(\frac{i}{n}\alpha_n^{\mathcal{M}} e^{-in\sigma_-}\right). \tag{2.66}$$

The case of $\tilde{\alpha}_n^{\mathcal{M}}$ is computed in the same way.

If the string is closed and winds around a compactified spatial direction, X^θ, of radius R, then

$$X^\theta(\tau, \sigma + 2\pi) = X^\theta(\tau, \sigma) + 2\pi R\omega, \tag{2.67}$$

where ω is the winding number. Therefore

$$\begin{aligned} X_+^\theta\left(\tau + (\sigma + 2\pi)\right) &= X_+^\theta(\sigma_+ + 2\pi) & (2.68)\\ &= \frac{x_+^\theta}{2} + \alpha' p_+^\theta(\sigma_+ + 2\pi) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\tilde{\alpha}^\theta}{n} e^{-in(\sigma_+ + 2\pi)} \\ &= X_+^\theta(\sigma_+) + 2\pi\alpha' p_+^\theta. & (2.69)\end{aligned}$$

In an analogous way,

$$X_-^\theta\left(\tau - (\sigma + 2\pi)\right) = X_-^\theta(\sigma_- - 2\pi) = X_-^\theta(\sigma_-) - 2\pi\alpha' p_-^\theta. \tag{2.70}$$

Therefore, substituting in (2.67):

$$2\pi\omega R = X^\theta(\tau, \sigma + 2\pi) - X^\theta(\tau, \sigma) = 2\pi\alpha'(p_+^\theta - p_-^\theta), \tag{2.71}$$

and consequently

$$p_+^\theta - p_-^\theta = \frac{\omega R}{\alpha'}. \tag{2.72}$$

Furthermore, the center of mass momentum is quantized due to the Kaluza-Klein compactification, so

$$p_+^\theta + p_-^\theta = \frac{n}{R}, \tag{2.73}$$

and therefore

$$\begin{aligned} p_-^\theta &= \frac{n}{R} - \frac{\omega R}{\alpha'}, & (2.74)\\ p_+^\theta &= \frac{n}{R} + \frac{\omega R}{\alpha'}. & (2.75)\end{aligned}$$

The spectrum of closed string states is determined by specific constraints and the mass formula. One of these constraints is the level matching condition, which ensures the consistency of the

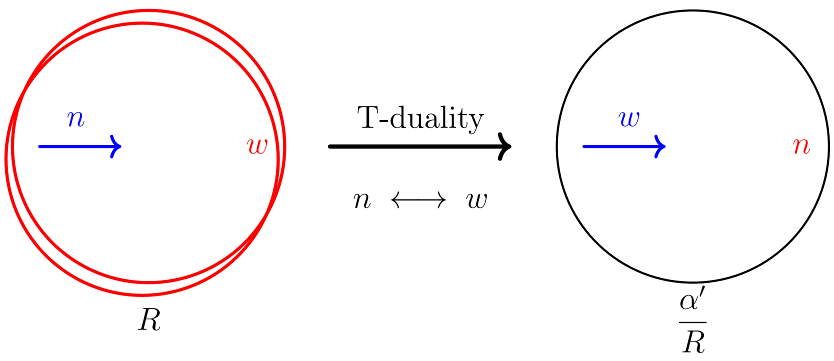

Figure 2.4: *T-duality: a bosonic string compactified on a circle of radius R is equivalent to one compactified on α′/R; winding states in the former correspond to Kaluza–Klein momentum states in the latter, and vice-versa.*

string periodic boundary conditions in compact dimensions. This condition is expressed as:

$$\bar{N} - N = n\omega, \tag{2.76}$$

where N and $\bar{N}$ are the excitation numbers of the left- and right-moving modes of the string, respectively.

A closed string compactified on a circle can both wind the circle and carry quantized momentum along it, so its mass receives separate energy contributions from the winding and the Kaluza-Klein momenta obtained above. In addition, quantum oscillations of the left- and right-moving modes contribute through their occupation numbers N and $\bar{N}$. Considering this, together with the Virasoro conditions, the mass of a closed string state is given by:

$$M^2 = \left(\frac{\omega R}{\alpha'}\right)^2 + \left(\frac{n}{R}\right)^2 + \frac{2}{\alpha'}(N + \bar{N} - 2). \tag{2.77}$$

The first term on the right hand side corresponds to the contribution from the winding number. This term grows with the radius R of the compactified dimension. The second term represents the contribution from the quatization of the momentum n, which becomes more relevant as the radius R decreases. The third term arises from the oscillatory excitations of the string. Substituting the level matching condition we obtain that

$$M^2 = \left(\frac{\omega R}{\alpha'}\right)^2 + \left(\frac{n}{R}\right)^2 + \frac{2}{\alpha'}(2\bar{N} - 2 - n\omega). \tag{2.78}$$

This formula shows the invariance of the mass spectrum when the radius of compactification transforms as $R \to \alpha'/R$, and the roles of n and ω are exchanged, leaving the mass spectrum unchanged, *i.e.*, the mass spectrum is invariant under the T-duality transformation:

$$n' = w, \quad w' = n, \quad R' = \alpha'/R. \tag{2.79}$$

Buscher's rules

Buscher [81] derived the explicit transformation rules for the closed string sector under T-duality, which applies to backgrounds with an isometry.

Assuming the existence of a isometry along a compact coordinate $\theta \sim \theta + 2\pi R$, and that the background fields do not depend on θ, the Buscher rules describe how the NS-NS fields transform under T-duality along θ. Writing the ten-dimensional coordinates as $X^{\mathcal{M}} = (X^\mu, X^\theta)$, where $\mu = 0, \ldots, D-2$ denote the non-compact directions, the dual fields $\tilde{g}_{\mathcal{M}\mathcal{N}}$, $\tilde{B}_{\mathcal{M}\mathcal{N}}$, and $\tilde{\Phi}$ are given by:

$$\tilde{g}_{\theta\theta} = \frac{1}{g_{\theta\theta}}, \qquad \tilde{g}_{\theta\mu} = \frac{B_{\theta\mu}}{g_{\theta\theta}}, \qquad \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{g_{\theta\mu}g_{\theta\nu} - B_{\theta\mu}B_{\theta\nu}}{g_{\theta\theta}}, \tag{2.80}$$

$$\tilde{B}_{\theta\mu} = \frac{g_{\theta\mu}}{g_{\theta\theta}}, \qquad \tilde{B}_{\mu\nu} = B_{\mu\nu} - \frac{g_{\theta\mu}B_{\theta\nu} - B_{\theta\mu}g_{\theta\nu}}{g_{\theta\theta}}, \tag{2.81}$$

$$\tilde{\Phi} = \Phi - \frac{1}{2}\ln g_{\theta\theta}. \tag{2.82}$$

These relations are known as Buscher's rules.

In addition to these rules, in the context of type IIA and type IIB there is a generalization of Buscher's rules that relates the R-R potentials of both theories [82, 83]. From type IIA to type IIB we have:

$$C^{(2n)}_{\mu_1\cdots\mu_{2n}} = C^{(2n+1)}_{\mu_1\cdots\mu_{2n}\theta} + 2nB_{[\mu_1|\theta|}C^{(2n-1)}_{\mu_2\cdots\mu_{2n}]} \tag{2.83}$$

$$- 2n(2n-1)\frac{B_{[\mu_1|\theta|}g_{\mu_2|\theta|}C^{(2n-1)}_{\mu_3\cdots\mu_{2n}]\theta}}{g_{\theta\theta}}, \tag{2.84}$$

$$C^{(2n)}_{\mu_1\cdots\mu_{2n-1}\theta} = -C^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}} + (2n-1)\frac{g_{[\mu_1|\theta|}C^{(2n-1)}_{\mu_2\cdots\mu_{2n-1}]\theta}}{g_{\theta\theta}}. \tag{2.85}$$

and from type IIB to type IIA:

$$C^{(2n+1)}_{\mu_1\cdots\mu_{2n+1}} = -C^{(2n+2)}_{\mu_1\cdots\mu_{2n+1}\theta} + (2n+1)B_{[\mu_1|\theta|}C^{(2n)}_{\mu_2\cdots\mu_{2n+1}]}$$

$$+ 2n(2n+1)\frac{B_{[\mu_1|\theta|}g_{\mu_2|\theta|}C^{(2n)}_{\mu_3\cdots\mu_{2n+1}]\theta}}{g_{\theta\theta}}, \tag{2.86}$$

$$C^{(2n+1)}_{\mu_1\cdots\mu_{2n}\theta} = C^{(2n)}_{\mu_1\cdots\mu_{2n}} + 2n\frac{g_{[\mu_1|\theta|}C^{(2n)}_{\mu_2\cdots\mu_{2n}]\theta}}{g_{\theta\theta}}. \tag{2.87}$$

2.3.2. S-duality

The S-duality is a duality between the strong and weak coupling regimes ($g_s \leftrightarrow \frac{1}{g_s}$). A paradigmatic case of this duality appears in type IIB string theory, which exhibits an $SL(2,\mathbb{Z})$ symmetry. In the context of the low-energy limit, in type IIB supergravity, the fields Φ and $C_{(0)}$, and $B_{(2)}$ and $C_{(2)}$ are paired under $SL(2,\mathbb{R})$. The R-R scalar $C_{(0)}$ and the dilaton Φ can be combined to define a complex scalar, known as the axio-dilaton τ, which is given by

$$\tau = C_{(0)} + ie^{-\Phi}. \tag{2.88}$$

The SL$(2,\mathbb{R})$ symmetry of the equations of motion acts on τ as a Möbius transformation:

$$\tau \to \frac{a\tau + b}{c\tau + d}, \tag{2.89}$$

where $ad - bc = 1$.

On the other hand, the R-R two-form potential $C_{(2)}$ and the Kalb-Ramond two-form $B_{(2)}$ transform under the SL$(2,\mathbb{R})$ symmetry as a doublet:

$$\begin{pmatrix} B_{(2)} \\ C_{(2)} \end{pmatrix} \to \begin{pmatrix} d & -c \\ -b & a \end{pmatrix} \begin{pmatrix} B_{(2)} \\ C_{(2)} \end{pmatrix}, \tag{2.90}$$

where a, b, c, and d are elements of the SL$(2,\mathbb{R})$ matrix that satisfy $ad - bc = 1$.

Considering the specific case in which the R-R scalar $C_{(0)}$ vanishes, we can take $a = d = 0$ and $b = -c = 1$, leading to the transformation $g_s = e^{\Phi} \to \frac{1}{g_s}$. Under this transformation, the fields transform as follows:

$$\Phi \to -\Phi, \qquad B_{(2)} \to C_{(2)}, \qquad C_{(2)} \to -B_{(2)}. \tag{2.91}$$

This transformation swaps the fields $B_{(2)}$ and $C_{(2)}$, essentially interchanging the roles of the fundamental string and the D1-brane in type IIB string theory. Note that fundamental strings couple to $B_{(2)}$ but not to $C_{(2)}$, while D1-branes are charged under $C_{(2)}$ but not under $B_{(2)}$. Therefore, S-duality relates these two configurations, illustrating a duality between fundamental strings and D1-branes under strong-weak coupling duality.

Another important example of this duality is the relationship between the Heterotic SO(32) and type I string theories. At the level of supergravity, their equivalence becomes evident by comparing their actions.

In the Einstein frame, the metrics of these theories are related by the dilaton field Φ as:

$$g^E_{MN} = e^{-\frac{1}{2}\Phi} g^S_{MN}, \tag{2.92}$$

where g^E_{MN} and g^S_{MN} are the Einstein frame and string frame metrics.

The bosonic parts of their actions in the Einstein frame are:

$$S^E_{\mathrm{I}} = \frac{1}{2\kappa^2_{10}} \int d^{10}x \sqrt{|g^E|} \left[R^E - \frac{1}{2}(\partial\Phi)^2 - \frac{e^{\Phi}}{12}|\tilde{F}_{(3)}|^2 - \frac{1}{4} e^{\frac{\Phi}{2}} \mathrm{Tr}_v(|\mathcal{F}_{(2)}|^2) \right], \tag{2.93}$$

$$S^E_{\mathrm{H}} = \frac{1}{2\kappa^2_{10}} \int d^{10}x \sqrt{|g^E|} \left[R^E - \frac{1}{2}(\partial\Phi)^2 - \frac{e^{-\Phi}}{12}|\tilde{H}_{(3)}|^2 - \frac{1}{4} e^{-\frac{\Phi}{2}} \mathrm{Tr}_v(|\mathcal{F}_{(2)}|^2) \right]. \tag{2.94}$$

The two actions are related by the transformations:

$$\Phi \to -\Phi, \quad \tilde{F}_{(3)} \to \tilde{H}_{(3)}. \tag{2.95}$$

When these actions are expressed in the string frame, the metrics and fields are related by:

$$\Phi \to -\Phi, \quad \tilde{F}_{(3)} \to \tilde{H}_{(3)}, \quad g_{\mathcal{MN}} \to e^{-\Phi} g_{\mathcal{MN}}. \tag{2.96}$$

Beyond the supergravity approximation, evidence for S-duality is also found in non-perturbative effects. For example, the tension[5] of a D1-brane in type I string theory is

$$T^{\mathrm{I}}_{\mathrm{D1}} = \frac{1}{g_s 2\pi (l_s)^2}, \tag{2.97}$$

while the tension of a fundamental string (F1-string) in Heterotic SO(32) theory is

$$T^{\mathrm{H}}_{\mathrm{F1}} = \frac{1}{2\pi (l_s)^2}. \tag{2.98}$$

Under S-duality, where $g_s \leftrightarrow 1/g_s$, these tensions are equivalent, providing further support for the correspondence between these theories.

[5]The tension of a Dp-brane, which amounts to the generalization of the string tension for objects exhibiting higher dimensional worldvolumes, will be defined in more detail in the next chapter.

Chapter 3

Charged objects in string theory

Understanding the low-energy dynamics of D-branes and O-planes is important to string theory, both for ensuring consistency and for applications to phenomenology. The effective action of D-branes, given by the Dirac-Born-Infeld and Wess-Zumino terms, describes the interaction between worldvolume gauge fields, background fields, and Ramond-Ramond charges. Orientifold planes, defined by discrete worldsheet symmetries, are essential for building consistent unoriented theories. Their presence alters the spectrum and gauge symmetries, especially when combined with D-branes, enabling compactifications with reduced supersymmetry.

Beyond their effective worldvolume description, D-branes and O-planes also admit a dual interpretation as classical solutions of ten-dimensional supergravity. Under this point of view, D-branes appear as BPS solitonic objects sourcing specific R-R fluxes and inducing curved spacetime geometries, while preserving part of the underlying supersymmetry. These supergravity solutions are essential for constructing flux compactifications and for realizing holographic dualities. In particular, the near-horizon geometry of D3-branes leads to the celebrated $\mathrm{AdS}_5 \times S^5$ background, which underpins the AdS/CFT correspondence and connects type IIB string theory to $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. This duality offers a profound bridge between gravitational and gauge descriptions. In this chapter, we develop these ideas, laying the groundwork for their applications in the next chapters. The content of this chapter is based on [60, 61, 62, 63, 64].

3.1 | D-brane effective action

As discussed in Section 2.1, D-branes are objects that impose boundary conditions on the endpoints of open strings. If the string is charged, its endpoints couple to a worldvolume gauge potential $\mathcal{A}$ with field strength $\mathcal{F} = d\mathcal{A}$. Let ξ^M, $M = 0, \ldots, p$, be the worldvolume coordinates, with $\xi^0 \equiv \tau$ the time coordinate and the remaining p spanning the spatial directions of the D-brane.

The bosonic part of a single Dp-brane action is given by the Dirac-Born-Infeld (DBI) action [84] plus a topological contribution, that is known as Wess-Zumino (WZ) term.

$$S_{\mathrm{D}p} = S_{\mathrm{D}p}^{\mathrm{DBI}} + S_{\mathrm{D}p}^{\mathrm{WZ}}, \tag{3.1}$$

where the DBI action is given by:

$$S_{\mathrm{D}p}^{\mathrm{DBI}} = -T_{\mathrm{D}p} \int d^{p+1}\xi \, e^{-\Phi} \sqrt{-\det\left(\mathrm{P}[g]_{MN} + \mathrm{P}[B_{(2)}]_{MN} + 2\pi\alpha' \mathcal{F}_{MN}\right)}, \tag{3.2}$$

$\mathcal{F} = d\mathcal{A}$ and $T_{\mathrm{D}p} = (2\pi)^{-p}\alpha'^{-(p+1)/2}$ is the tension of the Dp-brane. Here, P$[g]$ and P$[B]$ represent the pullbacks of the metric $g_{\mathcal{MN}}$ and the Kalb-Ramond two-form field $B_{\mathcal{MN}}$ onto the Dp-brane worldvolume, respectively.

The pullback of the metric $g_{\mathcal{MN}}$ onto the D-brane worldvolume is defined as

$$\mathrm{P}[g]_{MN} = \frac{\partial X^{\mathcal{M}}}{\partial \xi^{M}} \frac{\partial X^{\mathcal{N}}}{\partial \xi^{N}} g_{\mathcal{MN}}, \tag{3.3}$$

where $X^{\mathcal{M}}(\xi)$ are the embedding functions describing the location of the Dp-brane in the target space.

On the other hand, the Wess-Zumino term appearing in (3.1) encodes the coupling between the R-R potentials $C_{(q+1)}$ and the gauge invariant combination $P[B_{(2)}] + 2\pi\alpha'\mathcal{F}$ on the brane. It is given by [62]:

$$S^{\mathrm{WZ}}_{\mathrm{D}p} = \mu_p \int_{\Sigma_{p+1}} \sum_q \mathrm{P}[C_{(q+1)}] \wedge e^{\mathrm{P}[B_{(2)}]+2\pi\alpha'\mathcal{F}}. \tag{3.4}$$

A Dp-brane carries a positive charge $\mu_{\mathrm{D}p} \equiv T_{\mathrm{D}p}$ with respect to a RR $(p+1)$-form field. The corresponding anti-brane, denoted as $\overline{\mathrm{D}p}$, will have the same tension, but carry opposite charge.

For type IIA theory, the introduction of D-branes is restricted to those with even spatial dimensions, meaning that Dp-branes can exist only if p is even. On the other hand, the type IIB theory allows only Dp-branes with odd values of p [85].

3.2 | O-plane effective action

Besides Dp-branes, other charged objects are present in the spectrum, *i.e.* orientifold planes, denoted as O-planes. These play an important role in defining consistent string backgrounds that are free of anomalies. In turn, they impose some truncations that project out some of the fields of the theory. Consequently, they play an important role in the construction of unoriented string theories and models with reduced supersymmetry.

O-planes were introduced in [86], and are the loci of fixed points of a given orientifold $\mathbb{Z}_2$ action $\Omega_{\mathrm{O}p}$, which may be defined through

$$\Omega_{\mathrm{O}p} \;=\; \Omega\, \sigma_{\mathrm{O}p}\, \sigma_{F_L} \;, \tag{3.5}$$

where Ω is the worldsheet parity acting on the closed sector as

$$\Omega : \qquad \begin{array}{l} g_{MN} \;\to\; g_{MN} \;, \\ B_{MN} \;\to\; -B_{MN} \;, \\ \Phi \;\to\; \Phi \;, \end{array} \qquad C_{(k)} \;\to\; (-1)^{q+r} C_{(k)} \;, \tag{3.6}$$

where $k = 2q + r$, with $r = 0$ (type IIB), or $r = 1$ (type IIA). The second $\mathbb{Z}_2$ factor $\sigma_{\mathrm{O}p}$ is a spacetime involution flipping the sign of all transverse coordinates

$$\mathrm{O}p\ :\qquad \underbrace{\times\ \cdots\ \times}_{(p+1)\mathrm{D\ worldvolume}}\quad |\ \underbrace{-\ \cdots\ -}_{\mathrm{transverse}\ y^i}\ ,\qquad \sigma_{\mathrm{O}p}\ :\ y^i\ \longrightarrow\ -y^i\ .$$

Finally, σ_{F_L}, that involves the so-called fermion number that appear in the GSO projection [87][1], and is given by

$$\sigma_{F_L}\ =\ \begin{cases} (-1)^{F_L} & , \quad p = 2,3 \bmod 4\ , \\ 1 & , \quad p = 0,1 \bmod 4\ . \end{cases} \tag{3.7}$$

In the type IIA theory, O-planes, like D-branes, are restricted to even spatial dimensions. And the type IIB theory allows only Op-planes with odd values of p.

Depending on the sign of charge and tension, there exist four different types of Op-planes. The bosonic effective action of an O$p^{(\epsilon_1,\epsilon_2)}$-plane in type II string theory, with $\epsilon_1, \epsilon_2 \in \{-1,+1\}$, is

$$S_{\mathrm{O}p^{(\epsilon_1,\epsilon_2)}}\ =\ S^{\mathrm{DBI}}_{\mathrm{O}p^{(\epsilon_1,\epsilon_2)}}\ +\ S^{\mathrm{WZ}}_{\mathrm{O}p^{(\epsilon_1,\epsilon_2)}}\ , \tag{3.8}$$

where the DBI and WZ terms are

$$S^{\mathrm{DBI}}_{\mathrm{O}p^{(\epsilon_1,\epsilon_2)}} =\ -\,T_{\mathrm{O}p}\int d^{p+1}x\, e^{-\Phi}\sqrt{-\det(g_{MN})}\ , \tag{3.9}$$

$$S^{\mathrm{WZ}}_{\mathrm{O}p^{(\epsilon_1,\epsilon_2)}} = \mu_{\mathrm{O}p}\int\limits_{\mathrm{WV(O}p)} C_{(p+1)}\ , \tag{3.10}$$

with

$$T_{\mathrm{O}p} = \epsilon_1\, 2^{p-4} T_{\mathrm{D}p}\ , \qquad\qquad \mu_{\mathrm{O}p} = \epsilon_2\, 2^{p-4}\mu_{\mathrm{D}p}\ . \tag{3.11}$$

When charge and tension coincide, these O-planes are conventionally denoted by Op^+ & Op^- and their tension $T_{\mathrm{O}p^\pm}$ and charge $\mu_{\mathrm{O}p^\pm}$ are therefore:

$$T_{\mathrm{O}p^\pm}\ =\ \mu_{\mathrm{O}p^\pm}\ =\ \pm\, 2^{p-4}\, T_{\mathrm{D}p}\ . \tag{3.12}$$

An interesting example is the O9$^-$-plane in type IIB theory. This plane has a tension of $T_{\mathrm{O9}} = -32\, T_{\mathrm{D9}}$ and a corresponding charge related to the $C_{(10)}$ potential. This is important in the construction of type I string theory, as its presence ensures charge cancellation when combined with 32 D9-branes [88].

3.3 | Op/Dp systems

Each Dp-brane has a massless vector multiplet associated with the light open string state attached to it. Its low energy description is given by U(1) maximal SYM in $(p+1)$ dimensions.

[1]The GSO projection is a type of truncation that tunrs the RNS string theory into a consistent theory by projecting out a tachyon and preserving supersymmetry in 10D.

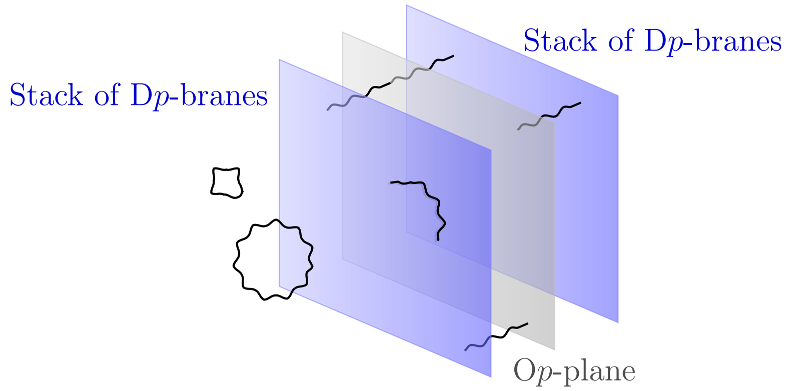

Figure 3.1: *Representation of a stack of Dp-branes and an Op-plane.*

Now, a set of N Dp-branes which are kept separated from one another at finite distance describes an Abelian $\mathrm{U}(1)^N$ gauge theory. However, in the limit where these are made to collide together to form a brane stack, the system undergoes a gauge symmetry enhancement to the non-Abelian gauge group $\mathrm{U}(N)$. In this case, the N^2 generators of $\mathrm{U}(N)$ are in one-to-one correspondence with light strings having each extremum on any D-brane within the stack. To study Dp-brane actions on curved backgrounds with fluxes we will follow [89, 90]. In this section we review some relevant aspects of the bosonic effective actions of an Op-plane and a stack of Dp-branes.

As mentioned above, open strings on a single Dp-brane are described by a $\mathrm{U}(1)$ gauge theory. However, if we consider a stack of N coincident Dp-branes, from the viewpoint of the string worldsheet, it is now possible to attach Chan-Paton factors to the endpoints of open strings. These factors correspond to fixed, non-dynamical degrees of freedom on the worldsheet, serving to label the strings according to the branes on which their endpoints lie. For instance, a Chan-Paton label λ_{ij} can be assigned to an open string stretching from brane i to brane j, with $i, j \in \{1, \dots, N\}$. In this way, the set of matrices λ associated to the strings can be regarded as forming a representation of a certain Lie algebra. In the case of oriented strings, the only compatible choice for this Lie algebra is $\mathrm{U}(N)$, where N is the number of coincident D-branes. In such a description, the Chan-Paton matrices λ can be taken to be Hermitian, with λ_{ij} representing their components. While these labels act as generators of a global $\mathrm{U}(N)$ symmetry on the worldsheet, in the spacetime picture this symmetry is promoted to a local gauge symmetry supported on the D-brane worldvolume. Therefore, the low-energy dynamics of open strings ending on coincident D-branes is described by a non-Abelian gauge theory.

When considering a system made out of Dp-branes and parallel Op-planes, in order to fully specify the dynamics, we also need to spell out the orientifold action on the open string states living on the D-branes. This is done by identifying its action on the open string Chan-Paton factors λ. For N Dp's and one Op [91], this reads

$$\lambda \overset{\Omega}{\to} M[\Omega]^{-1}\lambda^{\mathrm{T}} M[\Omega] \, , \qquad \text{with} \quad M[\Omega] \; = \left\{ \begin{array}{cl} \mathbb{I}_{2N} & \text{, for O}p^{-}, \\ \mathbb{J}_{2N} \equiv \begin{pmatrix} \mathbb{O}_N & i\,\mathbb{I}_N \\ -i\,\mathbb{I}_N & \mathbb{O}_N \end{pmatrix} & \text{, for O}p^{+}. \end{array} \right.$$

The above difference in the orientifold action at the level of the Chan-Paton factors results in

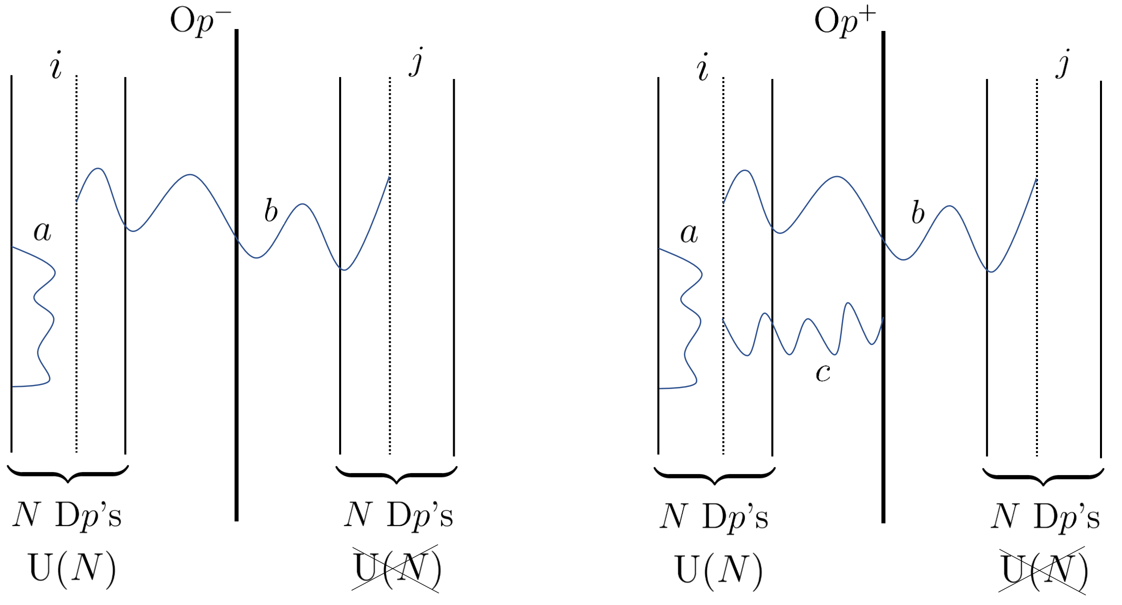

Figure 3.2: (Left) *In the presence of an* Op^-*, a stack of* N *coincident* Dp*-branes realizes an* SO($2N$) *gauge group. Its* $N(2N-1)$ *light dof's can be understood as all open strings with both extrema on one side of the O-plane (type a,* N^2 *states), plus those with one extremum on each side, with the Chan-Paton rule that* $i \neq j$ *(type b,* $N(N-1)$ *states).* (Right) *In the presence of an* Op^+*, we still have open strings of type a & b (these are now* N^2 *states due to the absence of the* $i \neq j$ *rule), and in addition we find strings connecting each D-brane to the O-plane (type c,* N *states). This yields a total of* $N(2N+1)$ *light states realizing* USp($2N$).

different open string SYM gauge groups in presence of an Op^+, or an Op^-. In the former case we have an USp($2N$) group, while in the latter we have SO($2N$) instead. The corresponding conceptual picture in these two situations can be found in Figure 3.2.

For the most general system made out of Dp-branes & Op-planes in the absence of fluxes and in flat space, the following tadpole cancellation condition is required for UV-finiteness of the corresponding quantum description

$$N_{\mathrm{D}p}\mu_{\mathrm{D}p} \ + \ N_{\overline{\mathrm{D}p}}\mu_{\overline{\mathrm{D}p}} \ + \ N_{\mathrm{O}p^+}\mu_{\mathrm{O}p^+} \ + \ N_{\mathrm{O}p^-}\mu_{\mathrm{O}p^-} \ \stackrel{!}{=} \ 0 \ , \tag{3.13}$$

which can be written as

$$\left(N_{\mathrm{D}p} \ - \ N_{\overline{\mathrm{D}p}}\right) \ \stackrel{!}{=} \ 2^{p-4}\left(N_{\mathrm{O}p^-} \ - \ N_{\mathrm{O}p^+}\right) \ , \tag{3.14}$$

where $N_{\mathrm{D}p} = 2N$ accounts for the imagine branes as well. It is crucial to remember that the above constraint originates from demanding that string amplitudes be free of divergences and it refers to amplitudes calculated in flat space and in the absence of fluxes. In Chapter 6 we will be considering more involved situations where the background fluxes may contribute in several ways to the tadpole constraints for the corresponding spacetime filling sources. We will therefore assume that the generalized versions of (3.14), despite not being shown in this manuscript, play an analogous role in guaranteeing UV-finiteness of string amplitudes.

If we now go back to the brane configurations obtained by setting $N_{\overline{\mathrm{D}p}} = 0$ in equation (3.14), we find that it is actually possible to have both Op^+'s & Op^-'s at the same time, as long as the constraint $N_{\mathrm{O}p^-} \geq N_{\mathrm{O}p^+}$ is respected, in such a way that tadpole cancellation is realized. In this situation, tadpole cancellation would simply require adding the following amount of parallel Dp-branes

$$N_{\mathrm{D}p} \stackrel{!}{=} 2^{p-4}\left(N_{\mathrm{O}p^-} - N_{\mathrm{O}p^+}\right) . \tag{3.15}$$

In this most general setup, if we furthermore allow for the possibility that these objects be separated into smaller groups from one another, we find that the most general gauge group will be of the form

$$G_{\mathrm{YM}} = \left(\prod_a \mathrm{U}(N_a)\right) \times \left(\prod_b \mathrm{SO}(2N_b)\right) \times \left(\prod_c \mathrm{USp}(2N_c)\right) . \tag{3.16}$$

The corresponding total number of massless vector fields reads

$$\mathfrak{N} \equiv \sum_a N_a^2 + \sum_b N_b(2N_b - 1) + \sum_c N_c(2N_c + 1) , \tag{3.17}$$

which will precisely coincide with the bare quantity appearing in the lower dimensional supergravity description. In Chapter 6, when discussing how these open string gaugings are embedded within the effective lower dimensional gauged supergravity theories, we will no longer specify an explicit form of the YM gauge group, nor specifically discuss concrete brane setup's.

In what follows we adopt the notation in [23]. We denote by x^M the worldvolume coordinates, whereas the transverse coordinates to the branes are called y^i. Therefore, the 10-dimensional spacetime coordinates $x^{\mathcal{M}}$ split into $x^{\mathcal{M}} = (x^M , y^i)$.

We will work in the *static gauge*, where the position of each brane reads $y^{Ii} = \lambda Y^{Ii}$, with $\lambda \equiv 2\pi\ell_s^2$, where ℓ_s denotes the string length and I runs in the number of branes. We will consider the generators of G_{YM} to live in the fundamental representation of the Lie algebra. Denoted by $\{t_I\}_{I=1,\ldots,\mathfrak{N}}$, they satisfy

$$[t_I, t_J] = i\, g_{IJ}{}^K t_K , \qquad \mathrm{Tr}(t_I t_J) = N \kappa_{IJ} , \tag{3.18}$$

where $\kappa_{IJ} = g_{IK}{}^L g_{JL}{}^K$ is the Cartan-Killing metric of G_{YM}.

The effective action governing the low-energy dynamics of bosonic fluctuations associated with a stack of N coinciding Dp-branes in type II string theory can be expressed as:

$$S_{\mathrm{D}p}^{\mathrm{bosonic}} = S_{\mathrm{D}p}^{\mathrm{DBI}} + S_{\mathrm{D}p}^{\mathrm{WZ}}, \tag{3.19}$$

where $S_{\mathrm{D}p}^{\mathrm{DBI}}$ represents the Dirac-Born-Infeld (DBI) term, which describes the dynamics of the branes under electromagnetic and gravitational influences, while $S_{\mathrm{D}p}^{\mathrm{WZ}}$ is the Wess-Zumino (WZ) term, capturing the coupling between the branes and background Ramond-Ramond fields.

The DBI part of the action is defined as follows:

$$S_{\mathrm{D}p}^{\mathrm{DBI}} = -T_{\mathrm{D}p} \int d^{p+1}x \,\mathrm{Str}\left(e^{-\hat{\Phi}} \sqrt{-\det(\mathbb{M}_{MN}) \det\left(\mathbb{Q}^i_j\right)} \right), \tag{3.20}$$

where $T_{\mathrm{D}p}$ is the brane tension, $\hat{\Phi}$ is the dilaton field, $\mathbb{M}_{MN}$ and $\mathbb{Q}^i_j$ are given by

$$\mathbb{M}_{MN} \equiv \mathrm{P}\left[\hat{E}_{MN} + \hat{E}_{Mi} \left(\mathbb{Q}^{-1} - \delta\right)^{ij} \hat{E}_{jN} \right] + \lambda \mathcal{F}_{MN}, \tag{3.21}$$

$$\mathbb{Q}^i_j \equiv \delta^i_j + i\lambda \left[Y^i, Y^k\right] \hat{E}_{kj}. \tag{3.22}$$

The combined background field $\hat{E}_{\mathcal{MN}}$, incorporating both the gravitational and Kalb-Ramond fields, is expressed as:

$$\hat{E}_{\mathcal{MN}} = \hat{g}_{\mathcal{MN}} + \hat{B}_{\mathcal{MN}}. \tag{3.23}$$

Here, $\hat{g}_{\mathcal{MN}}$ denotes the background spacetime metric, while $\hat{B}_{\mathcal{MN}}$ represents the Kalb-Ramond two-form field.

The hat symbol over these fields implies they are evaluated at the location of the Dp-branes $y^i = \lambda Y^i$, where $Y^i = Y^{Ii} t_I$ represents the transverse scalar fields. This position can be expanded using a Taylor series as follows:

$$\hat{\Phi}(x^M, \lambda Y^i) \equiv \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} Y^{i_1} \dots Y^{i_n} \partial_{i_1} \dots \partial_{i_n} \Phi(x^M, y^i)\Big|_{y^i=0}. \tag{3.24}$$

And $\mathcal{F}$ denotes the gauge field strength on the brane. On the Dp-brane, the field strength $\mathcal{F}$ of the gauge field $\mathcal{A}$ is defined as:

$$\mathcal{F} = d\mathcal{A} + i\mathcal{A} \wedge \mathcal{A} = \frac{1}{2} \mathcal{F}_{MN} dx^M \wedge dx^N, \tag{3.25}$$

where $\mathcal{F}_{MN}$ represents the components of the gauge field strength tensor on the worldvolume of the brane.

The P symbol refers to the pullback to the brane's worldvolume, where ordinary derivatives $\partial_M Y^i$ are substituted by the covariant derivative $D_M Y^i$:

$$D_M Y^i \equiv \partial_M Y^i + i[\mathcal{A}_M, Y^i], \tag{3.26}$$

where $\mathcal{A}_M$ is the gauge field associated with the brane. For example, the pullback of E_{MN} onto the brane is:

$$\mathrm{P}[\hat{E}_{MN}] = \hat{E}_{MN} + \lambda \hat{E}_{Mi} D_N Y^i + \lambda \hat{E}_{iN} D_M Y^i + \lambda^2 \hat{E}_{ij} D_M Y^i D_N Y^j. \tag{3.27}$$

And Str denotes the symmetrized trace. This operation implies that quantities such as Y^i in the Taylor expansion, as well as terms involving $\mathcal{F}_{MN}$, $D_M Y^i$, and the commutators $[Y^i, Y^j]$, are symmetrized before taking the trace.

On the other hand, the WZ part is expressed as:

$$S^{\mathrm{WZ}}_{\mathrm{D}p} = M_p \int \mathrm{Str}\left\{ \mathrm{P}\left(e^{i\lambda Y \iota_Y \iota_Y} \left[\sum_n \hat{C}_n \wedge e^{\hat{B}_2} \right) \right] \wedge e^{\lambda \mathcal{F}} \right\}. \tag{3.28}$$

where $\hat{C}_n$ are the Ramond-Ramond potentials.

Finally, the symbol ι_Y denotes the interior product with respect to the vector field Y^i. For instance,

$$\iota_Y \iota_Y \left(\frac{1}{2} C_{ij} dx^i \wedge dx^j \right) = -\frac{1}{2} C_{ij} [Y^i, Y^j]. \tag{3.29}$$

This operation effectively contracts the indices of the differential form with the components of Y, yielding terms involving commutators of Y^i.

3.4 | Supergravity solutions

An essential aspect of string theory is its low-energy limit, which is described by ten-dimensional type II supergravity. Within this effective theory, D-branes admit a dual interpretation: they are not only non-perturbative objects where open strings can end, but also classical solutions of the type IIA or type IIB supergravity theories. This perspective provides a bridge between the perturbative formulation of string theory and its gravitational, geometric description. In particular, D-branes emerge as BPS configurations preserving part of the supersymmetry and carrying Ramond-Ramond (R-R) charges, giving rise to curved spacetime geometries that reflect their presence.

Understanding D-branes as supergravity solutions is not merely a formal identification, it has profound implications. It allows one to construct flux compactifications and moduli stabilization scenarios in string phenomenology. Moreover, the explicit form of these solutions, and their behavior in different regimes (such as the near-horizon limit), plays a central role in formulating and testing conjectures like the AdS/CFT correspondence.

3.4.1. Dp-brane solutions

A Dp-brane is a solution in ten-dimensional supergravity that preserves half of the supersymmetry. Its worldvolume forms a flat hypersurface of $p+1$ dimensions, which remains invariant under the Poincaré group $\mathbb{R}^{p+1} \times \mathrm{SO}(1,p)$. The space perpendicular to the brane has dimension $9-p$.

For a Dp-brane in ten dimensions, the symmetries are $\mathbb{R}^{1,p} \times \mathrm{SO}(1,p) \times \mathrm{SO}(9-p)$. An ansatz that satisfies the equations of motion of type II supergravity is given by:

$$ds^2 = H_p(r)^{-1/2} \eta_{MN} dx^M dx^N + H_p(r)^{1/2} dy^i dy^i, \tag{3.30}$$

$$e^{\Phi} = g_s H_p(r)^{(3-p)/4}, \tag{3.31}$$

$$C_{(p+1)} = \left(H_p(r)^{-1} - 1 \right) dx^0 \wedge dx^1 \wedge \cdots \wedge dx^p, \tag{3.32}$$

$$B_{MN} = 0, \tag{3.33}$$

where x^M, with $M = 0, \dots, p$, denote the coordinates along the brane worldvolume, while y^i with $i = p+1, \dots, 9$ correspond to the spatial directions transverse to the brane. The radial coordinate r in the transverse space is given by

$$r^2 = \sum_{i=p+1}^{9} y_i^2. \tag{3.34}$$

Substituting this ansatz into the supergravity equations of motion implies that

$$\nabla^2 H_p(r) = 0. \tag{3.35}$$

So, for $r \neq 0$, the function $H_p(r)$ must satisfy the Laplace equation. The general spherically symmetric solution can therefore be expressed as

$$H_p(r) = 1 + \left(\frac{L_p}{r}\right)^{7-p}. \tag{3.36}$$

The constant 1 is chosen to ensure that, in the limit $r \to \infty$, the geometry asymptotically approaches ten-dimensional Minkowski space.

To fix the parameter L_p in the ansatz, one needs to evaluate the total R-R charge Q carried by the Dp-brane. This can be obtained by computing the flux of the $(p+2)$-form field strength over an $(8-p)$-dimensional sphere at spatial infinity, which encloses the source in the $(9-p)$-dimensional transverse space:

$$Q = \frac{1}{2\kappa_{10}^2} \int_{S^{8-p}} \star F_{(p+2)}. \tag{3.37}$$

where $\star$ denotes the ten-dimensional Hodge operator. The charge Q is related to the number N of coincident Dp-branes via $Q = N\mu_p$, where μ_p is the Dp-brane charge. By evaluating the integral and imposing $Q = N$, one finds the relation

$$L_p^{7-p} = (4\pi)^{(5-p)/2} \Gamma\left(\frac{7-p}{2}\right) g_s N \alpha'^{(7-p)/2}, \tag{3.38}$$

where Γ is the Euler Gamma function. In the special case of N coincident D3-branes, which play a central role in the AdS/CFT correspondence, one finds

$$L_3^4 = 4\pi g_s N \alpha'^2. \tag{3.39}$$

Near horizon limit of a D3-brane

Let us now focus on the case of a D3-brane. The geometry can be separated into two regimes: one at large distance ($r \gg L_3$) and another close to the brane ($r \ll L_3$). In the asymptotic region, $r \gg L_3$, equation (3.36) reduces to $H_3 \approx 1$, and the metric approaches to a flat ten-dimensional Minkowski spacetime. In contrast, in the near-horizon limit $r \ll L_3$, we have $H_3 \approx L_3^4/r^4$, so the line element becomes

$$ds^2 \approx \frac{r^2}{L_3^2}(-dt^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2) + \frac{L_3^2}{r^2}dr^2 + L_3^2 d\Omega_5^2, \tag{3.40}$$

and considering the coordinate transformation $r = L_3^2/x^0$, we will have

$$\begin{aligned} ds^2 &\approx \frac{L_3^2}{(x^0)^2}(-dt^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2) + (x^0)^2\frac{L_3^2}{(x^0)^4}(dx^0)^2 + L_3^2 d\Omega_5^2 \\ &= L_3^2\frac{-dt^2 + (dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2}{(x^0)^2} + L_3^2\, d\Omega_5^2, \end{aligned} \tag{3.41}$$

which is an $\mathrm{AdS}_5 \times S^5$ solution.

3.4.2. Supergravity Backgrounds of the Fundamental String (F1) and the NS5-Brane

Apart from the classical supergravity backgrounds generated by D-branes, one can also consider other supergravity solutions. In particular, let us focus on solutions carrying charge with respect to the Kalb-Ramond two-form field $B_{(2)}$, namely the fundamental string (F1) and its magnetic dual, the NS5-brane. Both of these arise in type IIA, type IIB, and heterotic supergravity theories, since they couple exclusively to the NS-NS sector.

In the Einstein frame, the fundamental string solution is represented by:

$$ds^2 = H_1(r)^{-3/4}\eta_{MN}dx^M dx^N + H_1(r)^{1/4}(dr^2 + r^2 d\Omega_7^2), \tag{3.42}$$

$$e^\Phi = H_1(r)^{-1/2}g_s, \tag{3.43}$$

$$B_{(2)} = \big(H_1(r)^{-1} - 1\big)\, dx^0 \wedge dx^1, \tag{3.44}$$

where the function $H_1(r)$ is defined as

$$H_1(r) = 1 + \frac{L^6}{r^6}, \quad L^6 = 32\pi^2 g_s^2 \alpha'^3. \tag{3.45}$$

The magnetic counterpart of the fundamental string is the NS5-brane, whose supergravity description in the Einstein frame takes the form

$$ds^2 = H_5(r)^{-1/4}\eta_{MN}dx^M dx^N + H_5(r)^{3/4}(dr^2 + r^2 d\Omega_3^2), \tag{3.46}$$

$$e^\Phi = H_5(r)^{1/2}g_s, \tag{3.47}$$

$$B_{(6)} = \big(H_5(r)^{-1} - 1\big)\, dx^0 \wedge \cdots \wedge dx^5, \tag{3.48}$$

where the potential $B_{(6)}$ is such that $dB_{(6)} = \star H_{(3)}$, and the function $H_5(r)$ is

$$H_5(r) = 1 + \frac{L^2}{r^2}, \quad L^2 = N\alpha', \tag{3.49}$$

with N denoting the number of NS5-branes in the configuration.

3.5 | AdS/CFT correspondence

In 1997, Maldacena formulated a correspondence between string theory and a quantum field theory. The conjecture states that type IIB string theory on a ten-dimensional background $\mathrm{AdS}_5 \times S^5$ is equivalent to a four-dimensional $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory. This equivalence is motivated by the fact that a stack of N coincident D3-branes can be described in two different ways.

In one description, the D3-branes are placed in a flat ten-dimensional Minkowski spacetime $\mathbb{R}^{1,9}$, with their worldvolume extending along the coordinates x^0, x^1, x^2, and x^3:

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D3	×	×	×	×	−	−	−	−	−	−

Table 3.1: *The symbol × denotes the directions along which the brane extends, while − indicates the directions in which the brane does not extend.*

In the low-energy limit, the theory only contains massless modes. The effective description then involves the massless states of both closed and open strings, as well as any additional massless states produced by their interactions. Therefore, the action can be expressed as

$$S = S_{\text{closed}} + S_{\text{open}} + S_{\text{int}}. \tag{3.50}$$

The term S_{open} comes from the excitations of N coincident D3-branes, which produce the massless states of open strings. For a single D3-brane, the low-energy field content consists of six scalar fields ϕ^i, the Kalb-Ramond fild $B_{\mathcal{MN}}$, a gauge field A_M, and a spinorial field. These fields are described, as we saw in (3.2), by the Dirac-Born-Infeld (DBI) action:

$$S_{\text{DBI}} = -\frac{1}{(2\pi)^3 g_s l_s^4} \int d^4x\, e^{-\phi} \sqrt{-\det\left(P[g]_{MN} + 2\pi\alpha' \mathcal{F}_{MN}\right)}, \tag{3.51}$$

where the Kalb-Ramond field is set to zero for simplicity. At low energies, $\mathcal{F} = d\mathcal{A}$ for a single D3-brane, and the action simplifies to

$$S = -\frac{1}{4\, g_{YM}^2} \int d^4x \left(\mathcal{F}_{MN}\mathcal{F}^{MN} + \mathcal{O}(\alpha')\right), \tag{3.52}$$

which corresponds to a U(1) Yang–Mills theory, with $g_{YM}^2 = 2\pi g_s$.

To generalize this to N coincident D3-branes, the open strings can have endpoints on any pair of branes in the stack. As a result, the gauge symmetry becomes U(N), with scalar and gauge fields transforming in the adjoint representation: $\phi^i = \phi^{ia} T_a$, $\mathcal{A}_M = \mathcal{A}_M^a T_a$. The kinetic term is promoted to $\mathcal{F}_{MN}^a \mathcal{F}^{aMN}$ to maintain gauge invariance. Taking the limit $\alpha' \to 0$, the action S_{open} reduces to the bosonic part of $\mathcal{N} = 4$ super Yang–Mills theory. The $U(N)$ gauge group decomposes as U(N) = SU$(N) \times$ U(1), where the U(1) part corresponds to the degrees of freedom associated with the center of mass of the brane system. These degrees of freedom decouple from the others, and one just consider the SU(N) part.

With respect to the interacting part, one can show that the interaction term is $S_{\rm int} \sim g_s \alpha'^2$, and therefore vanishes in the low-energy limit [92].

Finally, as we mention in the previous chapter, the closed string sector is described at low energies by type IIB supergravity.

The full system can be described in terms of two decoupled components: on the one hand, type IIB supergravity in ten dimensions. And on the other hand, the gauge theory on the worldvolume of the D3-branes. The open string excitations ending on the D3-branes form an $\mathcal{N} = 4$ vector multiplet, and the resulting effective theory is the four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory with gauge group SU(N).

From another perspective, the N D3-branes can be interpreted as classical solutions to the type IIB supergravity equations of motion. In this setup, the D3-branes act as sources for the Ramond-Ramond 5-form flux and carry both charge and tension. The corresponding supergravity solution is given by:

$$ds^2 = \frac{1}{\sqrt{H(r)}} \eta_{MN} dx^M dx^N + \sqrt{H(r)} \left(dr^2 + r^2 d\Omega_5^2 \right) , \tag{3.53}$$

with

$$H(r) = 1 + \left(\frac{L}{r} \right)^4 , \quad L^4 = 4\pi g_s N l_s^4. \tag{3.54}$$

This background contains a constant dilaton together with a self-dual Ramond-Ramond 5-form flux $F_{(5)}$. The geometry has two asymptotic regions, determined by the radial coordinate r. For $r \gg L$, the harmonic function approaches $H(r) \simeq 1$, and the metric tends to flat ten-dimensional Minkowski space-time. In the opposite limit, $r \ll L$, the background becomes asymptotically AdS:

$$ds^2 = \frac{r^2}{L^2} \eta_{MN} dx^M dx^N + \frac{L^2}{r^2} (dr^2 + r^2 d\Omega_5^2) = \frac{L^2}{r^2} \left(\eta_{MN} dx^M dx^N + dz^2 \right) + L^2 d\Omega_5^2 . \tag{3.55}$$

This expression is obtained after the coordinate transformation $z = L^2/r$. The resulting space-time has the structure of AdS$_5 \times S^5$, with both factors with the same radius of curvature L.

The excitations of the 3-brane are associated with closed string modes that propagate in a ten-dimensional spacetime which is asymptotically flat, as well as with closed strings in the near-horizon region. In the low-energy limit, these two sectors are decoupled. This decoupling can be illustrated by considering a string mode of energy E_r evaluated at a fixed radial coordinate close to the region $r = 0$. The corresponding energy as perceived by an observer at infinity, E_∞, is given by

$$E_\infty = H^{-1/4} E_r . \tag{3.56}$$

For a fixed value of E_r, taking the limit $r \to 0$ leads to $E_\infty \to 0$, showing that an observer located at infinity would only detect low-energy excitations. From this viewpoint, the two decoupled low-energy regimes observed at infinity are: on one side, fluctuations of type IIB supergravity in flat spacetime; and on the other side, closed string modes near $r = 0$, corresponding to the AdS$_5 \times$ S^5 geometry. At sufficiently low energies, these two regimes are decoupled.

The stack of N D3-branes admits two different descriptions. At low energies both lead to type IIB supergravity in ten dimensions and another decoupled sector. By identifying these remaining sectors, one is led to conjecture a duality: $\mathcal{N} = 4$ super Yang-Mills theory with gauge group SU(N) is dual to type IIB string theory formulated on an $\text{AdS}_5 \times \text{S}^5$ background. This is the statement of the Maldacena conjecture. The original formulation has been generalized to a broader class of dualities relating conformal field theories in p dimensions and string theories on backgrounds of the form $\text{AdS}_{p+1} \times \text{M}^{9-p}$.

3.6 | SUSY projectors of type IIB objects

Supersymmetry projectors are a key tool in studying the amount of preserved supersymmetry in configurations involving BPS objects. This approach is particularly useful when dealing with intersecting branes or other extended objects, as the preserved supersymmetries correspond to the common eigenspinors of all involved projectors. Therefore, supersymmetry projectors provide a simple and efficient method to determine the structure and fraction of preserved supersymmetry in a given setup.

In $(1+9)$D, spinors admit a Majorana-Weyl (MW) representation in which the Dirac matrices are realized as

$$\Gamma^M \;=\; \left(\begin{array}{c|c} 0_{16} & \Sigma^M \\ \overline{\Sigma}^M & 0_{16} \end{array}\right) \,, \tag{3.57}$$

where the 16×16 blocks Σ^M & $\overline{\Sigma}^M$ are real and act on chiral spinors. Since the 32 real supercharges of type IIB supergravity are represented by two distinct MW spinors of the same chirality, the same degrees of freedom can be rearranged into a single *complexified* chiral spinor

$$\epsilon \;=\; \zeta + i\,\eta \,, \tag{3.58}$$

where both ζ and η are MW. In terms of such complex MW spinor, $\frac{1}{2}$-BPS objects admit sets of Killing spinors spanned by the eigenspinors of a projection operator

$$\Pi(\mathcal{O}) \;\equiv\; \frac{1}{2}(\mathbb{I} + \mathcal{O}) \,, \tag{3.59}$$

where the operator $\mathcal{O}$ is an involution acting on complex spinors such as ϵ. The different SUSY projectors for the various fundamental BPS objects in type IIB supergravity are summarized in Table 3.2.

object	F1	NS5	W_B	KK5_B	D1	D3	D5	D7	D9
$\mathcal{O}$	$\Sigma^{01} \circ *$	$\Sigma^{6789} \circ *$	Σ^{01}	Σ^{6789}	$i\Sigma^{01} \circ *$	$i\Sigma^{0123}$	$i\Sigma^{6789} \circ *$	$i\Sigma^{89}$	$i*$

Table 3.2: *The different operators $\mathcal{O}$ appearing in the SUSY projector defined in* (3.59) *for the various fundamental BPS objects of type IIB string theory. We introduced the notation* $\Sigma^{i_1 \dots i_{2p}} \equiv \Sigma^{[i_1} \dots \overline{\Sigma}^{i_{2p}]}$*, while $*$ denotes complex conjugation. All objects are assumed to fill the first* $(p+1)$ *spacetime directions.*

When studying intersections of fundamental BPS objects, one has to find a set of common eigenspinors simultaneously preserved by all projectors corresponding to the various objects involved. Note that this will only be possible if the aforementioned operators commute. The (real) dimension of the common eigenspace will then represent the number of preserved supercharges.

Chapter 4

Gaugings and their higher dimensional origin

In supergravity theories, the notion of gauging refers to the promotion of a subset of global symmetries to local ones, thereby inducing non-trivial interactions among the fields and leading to the generation of scalar potentials and mass terms. To fully understand the gauging procedure, it is important to first consider the ungauged theories. These ungauged supergravities, which often arise as consistent truncations of higher-dimensional theories, serve as the base upon which the gauging is later implemented. In this context, the global symmetries play a dual role: they constrain the dynamics of the scalar fields and, simultaneously, provide the symmetry structure that can be gauged.

This chapter describes the construction of gauged supergravity theories and their relation to higher-dimensional origins. We will begin with an overview of ungauged supergravities, including the super-Poincaré algebra and the role of scalar fields, which parametrize a coset manifold G/H. Then, the embedding tensor formalism is introduced to define consistent gaugings of subgroups of the global symmetry group G, subject to linear and quadratic constraints. The discussion continues with the compactifications on group manifolds, where the structure constants appearing in the lower-dimensional theory are the objects that encode the information of the internal space. A reduction Ansatz is employed to derive the lower-dimensional action. The chapter also includes a dimensional reduction of type II supergravity theories, identifying the contributions of fluxes and curvature to the scalar potential. Finally, the role of localized sources is considered, presenting the structure of tadpole cancellation conditions and the Green–Schwarz mechanism for anomaly cancellation. The content of this chapter is based on [93, 94, 95, 60, 96].

4.1 | Ungauged supergravities

Let us consider a D-dimensional Minkowski spacetime with metric $\eta_{\mu\nu}$ and coordinates x^μ, and $\mu = 0, 1, \ldots, D-1$; its isometry group is the Poincaré group, and its associated Lie algebra, known as the Poincaré algebra, consists of the generators of infinitesimal translations, P_μ, and infinitesimal Lorentz transformations, $M_{\mu\nu}$. Their commutation relations are:

$$[M_{\mu\nu}, M_{\sigma\rho}] = \eta_{\nu\sigma} M_{\mu\rho} - \eta_{\mu\sigma} M_{\nu\rho} + \eta_{\mu\rho} M_{\nu\sigma} - \eta_{\nu\rho} M_{\mu\sigma}, \tag{4.1}$$

$$[P_\mu, P_\nu] = 0, \tag{4.2}$$

$$[M_{\mu\nu}, P_\rho] = \eta_{\nu\rho} P_\mu - \eta_{\mu\rho} P_\nu. \tag{4.3}$$

While this purely bosonic algebra encodes the kinematical symmetries of relativistic field theory, it does not relate states of integer and half-integer spin.

Supersymmetry provides a controlled and essentially unique extension which remedies this limitation by introducing transformations that relate particles of integer and half-integer spin. [1] The goal is now to expand the Poincaré algebra to include supersymmetry, to do this we enlarge the Minkowski spacetime with anticommuting coordinates ψ^α, where α is a spacetime spinor index, apart from the bosonic previous ones, x^μ, in what is called the superspace. Supersymmetry can be understood as the invariance of the theory under transformations that relate the bosonic coordinates and the fermionic coordinates in superspace. This leads to the appearance of fermionic symmetry generators of this kind of symmetries known as *supercharges*, denoted as Q_α. When there are multiple such fermionic directions, one obtains several sets of supercharges, labeled as Q^I_α, where $I = 1, 2, \ldots, \mathcal{N}$ indexes the different sets of fermionic coordinates.

The structure of the (anti)commutation relations among these generators depends on the spacetime dimension and the number of supersymmetries $\mathcal{N}$. For example, in four spacetime dimensions with $\mathcal{N} = 1$, adopting the conventions

$$\{\gamma^\mu, \gamma^\nu\} = 2\,\eta^{\mu\nu}, \quad \gamma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu], \quad \bar{Q}_\beta = {Q_\beta}^\dagger \gamma^0, \tag{4.4}$$

the relations are:

$$\{Q_\alpha, \bar{Q}_\beta\} = -\frac{1}{2} P_\mu (\gamma^\mu)_{\alpha\beta}, \quad [Q_\alpha, P_\mu] = 0, \quad [M_{\mu\nu}, Q_\alpha] = -\frac{1}{2} (\gamma_{\mu\nu})_\alpha{}^\beta Q_\beta, \tag{4.5}$$

where γ^μ are the gamma matrices in four dimensions, and the antisymmetrized product of gamma matrices is given by:

$$\gamma_{\mu_1 \ldots \mu_n} = \gamma_{[\mu_1} \gamma_{\mu_2} \cdots \gamma_{\mu_n]}. \tag{4.6}$$

This algebraic structure is known as the *super-Poincaré algebra*. Supergravity theories emerge when this algebra is gauged, much like how General Relativity can be seen as a gauge theory of the Poincaré algebra. Ungauged supergravity theories are those in which the gauging is limited to the super-Poincaré algebra itself, without promoting any additional internal global symmetries to local gauge symmetries.

The simplest nontrivial example of an ungauged supergravity theory is $\mathcal{N} = 1$ supergravity

[1] The appearance of supersymmetry as the (essentially) unique nontrivial extension of spacetime symmetries is tightly constrained by structural results in axiomatic S-matrix theory. The Coleman-Mandula theorem [124] shows that, under standard assumptions (existence of a nontrivial analytic S-matrix, a finite number of particle types below any mass, cluster decomposition, etc.), any nontrivial combination of internal and spacetime symmetries within an ordinary Lie-algebraic framework must be a direct product of the two, thereby forbidding nontrivial mixing.

Supersymmetry circumvents this no-go result by relaxing the hypothesis that all symmetry generators commute (*i.e.*, by admitting odd generators and anticommutators). The Haag–Lopuszanski–Sohnius theorem [123] makes this possibility precise and classifies the admissible extensions: allowing for spinorial generators, the only consistent enlargement of the Poincaré algebra that yields nontrivial mixing with internal symmetries is the supersymmetry algebra (possibly augmented by central/tensorial charges and an associated R-symmetry). That is to say, supersymmetry constitutes the unique, highly constrained mechanism to relate bosons and fermions while preserving the fundamental axioms of relativistic quantum field theory.

in four dimensions. Its gauge multiplet contains the vierbein $e_\mu{}^a$, where as usual a denotes a flat index, and a Majorana gravitino ψ_μ, which is the spin-3/2 gauge field associated to local supersymmetry. The on-shell action is given by the Einstein–Hilbert term in the second order formalism plus the covariantized Rarita–Schwinger term [95]:

$$S = \frac{1}{2\kappa^2}\left(\int d^4x\, e\, e^{a\mu}\, e^{b\nu}\, R_{\mu\nu ab}(\omega) - \int d^4x\, e\, \bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho\right), \tag{4.7}$$

where $\kappa = 8\pi G$ is the gravitational coupling constant in four dimensions, $e = \det e_\mu{}^a = \sqrt{-g}$, $R_{\mu\nu ab}$ is defined as

$$R_{\mu\nu ab} = \partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} + \omega_{\mu ac}\, \omega_\nu{}^c{}_b - \omega_{\nu ac}\, \omega_\mu{}^c{}_b, \tag{4.8}$$

and D_ν is the spacetime covariant derivative including the spin connection

$$D_\nu \psi_\rho = \partial_\nu \psi_\rho + \frac{1}{4}\omega_{\nu ab}\gamma^{ab}\psi_\rho. \tag{4.9}$$

Here $\omega_{\mu ab}$ is the torsion-free spin connection, defined by

$$\omega_\mu{}^{ab}(e) = 2\, e^{\mu[a}\partial_{[\mu} e_{\mu]}{}^{b]} - e^{\mu[a} e^{b]\sigma} e_{\mu c}\partial_\mu e_\sigma{}^c. \tag{4.10}$$

The local supersymmetry transformations that leave this action invariant (up to total derivatives) are

$$\delta e_\mu{}^a = \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu, \qquad \delta\psi_\mu = D_\mu \epsilon, \tag{4.11}$$

with $\epsilon(x)$ a local Majorana spinor.

4.1.1. Supergravities in lower dimensions

Supergravities with 32 supercharges are referred to as *maximally supersymmetric theories*, while those containing 16 are named *half-maximal supergravities*. When $\mathcal{N} = 1$ in dimension D, such models are commonly regarded as *minimal*, although the actual number of supercharges associated with minimal theories varies with D. This number increases with the spacetime dimension, culminating at $D = 11$, where the minimal and maximal formulations become identical. A more exhaustive classification of supergravity theories across different dimensions is available in references such as [97, 98, 99, 100, 101], and can be summarized in Table 4.1.

4.1.2. Scalar Manifold

The bosonic[2] field content of ungauged supergravity in D dimensions includes the metric $g_{\mu\nu}$, where Greek indices correspond to spacetime coordinates, a set of n_s scalar fields ϕ^i, with $i = 1, \ldots, n_s$, and n_v vector fields A^M_μ, with $M = 1, \ldots, n_v$. Additionally, the theory includes antisymmetric tensor fields $B^I_{\nu_1 \ldots \nu_p}$ of various ranks p, where I labels different antisymmetric

[2]The fermionic part of the action will be fully specified by supersymmetry once the bosonic part has been established.

D	**Supergravities ($\mathcal{N}$)**	**Number of supercharges**
11	1	32
10	(1,0) $\equiv$ I, (1,1) $\equiv$ IIA, (2,0) $\equiv$ IIB	16, 32, 32
9, 8, 7	1, 2	16, 32
6	(1,0) $\equiv$ i, (1,1) $\equiv$ iia, (2,0) $\equiv$ iib, (2,1), (3,0), (2,2), (3,1), (4,0)	8, 16, 16, 24, 24, 32, 32, 32
5	1, 2, 3, 4	8, 16, 24, 32
4	1, 2, 3, 4, 5, 6, 8	4, 8, 12, 16, 20, 24, 32

Table 4.1: *Classification of supergravity theories by dimension and supersymmetry*

forms present in the theory. The dynamics of these fields are governed by a Lagrangian of the form:

$$\mathcal{L}_{\text{bos}} = e\left(\frac{1}{2}\mathcal{R} - \frac{1}{2}G_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j - \frac{1}{4}M_{MN}(\phi)F^M_{\mu\nu}F^{N\mu\nu} + \cdots\right), \tag{4.12}$$

where $\mathcal{R}$ is the Ricci scalar and the field strengths $F^M_{\mu\nu}$ are defined as $F^M_{\mu\nu} \equiv \partial_\mu A^M_\nu - \partial_\nu A^M_\mu$. Ellipses denote the kinetic terms for the higher-rank forms and potential topological contributions.

The scalar fields ϕ^i serve as local coordinates of a non-compact, differentiable Riemannian manifold endowed with a positive definite metric $G_{ij}(\phi)$, this manifold is known as *scalar manifold*, and we will denote it as $\mathcal{M}_{\text{scalar}}$. The kinetic term in the bosonic Lagrangian results from the pullback of this metric via the map defined by ϕ^i:

$$\phi : \mathcal{M} \to \mathcal{M}_{\text{scalar}}, \quad x \mapsto \phi(x) = \left(\phi^1(x), \ldots, \phi^{n_s}(x)\right), \tag{4.13}$$

where $\mathcal{M}$ is the spacetime manifold and $\mathcal{M}_{\text{scalar}}$ is the scalar manifold. This leads to a description of the dynamics of the scalar fields through a non-linear sigma model. The kinetic term of the Lagrangian density is therefore expressed as:

$$\mathcal{L}_{\text{scalar}} = \frac{e}{2}G_{ij}(\phi)\,\partial_\mu\phi^i\partial^\mu\phi^j. \tag{4.14}$$

It is clear that the non-linear σ-model remains unchanged under global symmetries of the scalar field manifold. Therefore, the isometry group G of the scalar manifold defines the global symmetries of the scalar action.

$$G_{ij}(\phi')\,\partial_\mu\phi'^i\partial^\mu\phi'^j = G_{ij}(\phi)\,\partial_\mu\phi^i\partial^\mu\phi^j. \tag{4.15}$$

The isotropy group H of the scalar manifold can be generally expressed as $H = H_R \times H_{\text{matt}}$, where H_R denotes the automorphism group of the supersymmetry algebra, commonly referred to as the R-symmetry group, and H_{matt} is a compact group that acts on the matter fields. For $\mathcal{N} > 2$ we have that the scalar manifold is both homogeneous and symmetric and therefore:

$$\mathcal{M}_{\text{scalar}} = \frac{G}{H},$$

where G is the semisimple non-compact Lie group of isometries. The isotropy group H in this context is its maximal compact subgroup. In (half-)maximal supergravity theories, the scalar fields ϕ^i are represented within a G/H coset space. In Table 4.2 we show the global symmetry groups and their maximal compact subgroups for maximal and half-maximal supergravities in various dimensions.

D	G_{max}	H_{max}	$G_{\text{half-max}}$	$H_{\text{half-max}}$
9	GL(2)	SO(2)	$\text{GL}(1)\times\text{SO}(1,1+n)$	$\text{SO}(1+n)$
8	$\text{SL}(2)\times\text{SL}(3)$	$\text{SO}(2)\times\text{SO}(3)$	$\text{GL}(1)\times\text{SO}(2,2+n)$	$\text{SO}(2)\times\text{SO}(2+n)$
7	SL(5)	SO(5)	$\text{GL}(1)\times\text{SO}(3,3+n)$	$\text{SO}(3)\times\text{SO}(3+n)$
6	SO(5, 5)	$\text{SO}(5)\times\text{SO}(5)$	$\text{GL}(1)\times\text{SO}(4,4+n)$	$\text{SO}(4)\times\text{SO}(4+n)$
5	$E_{6(6)}$	USp(8)	$\text{GL}(1)\times\text{SO}(5,5+n)$	$\text{SO}(5)\times\text{SO}(5+n)$
4	$E_{7(7)}$	SU(8)	$\text{SL}(2)\times\text{SO}(6,6+n)$	$\text{SO}(2)\times\text{SO}(6)\times\text{SO}(6+n)$
3	$E_{8(8)}$	SO(16)	$\text{SO}(8,8+n)$	$\text{SO}(8)\times\text{SO}(8+n)$

Table 4.2: *List of global symmetry groups and their maximal compact subgroups arising in maximal and half-maximal supergravity theories in different spacetime dimensions.*

A convenient approach to formulate the previous sigma model consists of representing the scalar fields via a G-valued matrix $\mathcal{V}$ (evaluated in some fundamental representation of G), and using the left-invariant current J_μ defined as

$$J_\mu = \mathcal{V}^{-1}\partial_\mu\mathcal{V} \in \mathfrak{g}, \tag{4.16}$$

where $\mathfrak{g} \equiv \text{Lie } G$, such that $\mathcal{L}_{\text{scalar}} = -\frac{e}{2}\text{Tr}(J_\mu J^\mu)$. This Lagrangian is invariant under global G and local H transformations, which act on the scalar matrix $\mathcal{V}$ as follows:

$$\delta\mathcal{V} = \Lambda\mathcal{V} - \mathcal{V}h(x), \quad \Lambda \in \mathfrak{g}, \quad h(x) \in \mathfrak{h}. \tag{4.17}$$

Then J_μ transforms as:

$$\delta J_\mu = -\partial_\mu h(x) + [h(x), J_\mu]. \tag{4.18}$$

To respect the structure of the coset space, J_μ is split into

$$J_\mu = Q_\mu + P_\mu, \quad Q_\mu \in \mathfrak{h}, \quad P_\mu \in \mathfrak{p}, \tag{4.19}$$

where $\mathfrak{h} \equiv \text{Lie } H$ and $\mathfrak{p}$ is its complement, i.e., $\mathfrak{g} = \mathfrak{h} \perp \mathfrak{p}$, orthogonal with respect to the Cartan-Killing form. So the variations of P_μ and Q_μ are

$$\delta P_\mu = [h(x), P_\mu], \tag{4.20}$$
$$\delta Q_\mu = -\partial_\mu h(x) + [h(x), Q_\mu]. \tag{4.21}$$

The scalar Lagrangian can be expressed as

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2}e\,\text{Tr}(P_\mu P^\mu)\ , \tag{4.22}$$

where e is determinant of the vielbein.

The global $\mathfrak{g}$ transformations can be expressed as $\Lambda = \Lambda^\alpha t_\alpha$, expanded in a basis of generators t_α that satisfy the usual Lie algebra commutation relations

$$[t_\alpha, t_\beta] = \omega_{\alpha\beta}{}^\gamma\, t_\gamma, \tag{4.23}$$

where Λ^α are the constant parameters, $\omega_{\alpha\beta}{}^\gamma$ are the structure constants and $\alpha = 1, \ldots, \dim G$.

To clarify this, it is beneficial to express equation (4.17) in terms of indices as follows:

$$\delta \mathcal{V}_M{}^{\underline{N}} = \Lambda^\alpha (t_\alpha)_M{}^K \mathcal{V}_K{}^{\underline{N}} - \mathcal{V}_M{}^{\underline{K}} h_{\underline{K}}{}^{\underline{N}}, \tag{4.24}$$

where t_α are the generators of G, and the underlined indices $\underline{K}, \underline{N}$ indicate their transformation properties under the subgroup H.

In order to construct the full supersymmetric action, it is generally convenient to keep the local H gauge freedom. However, when one restricts attention to the purely bosonic part of the theory, it is useful to express the theory in terms of H-invariant quantities. A typical example is provided by the scalar fields, which may be encoded through a symmetric, positive-definite matrix $\mathcal{M}$ defined by

$$\mathcal{M} = \mathcal{V}\,\Delta\,\mathcal{V}^{\mathrm{T}}, \tag{4.25}$$

where Δ denotes a constant, positive-definite matrix invariant under the action of H (for example, for the coset $\mathrm{SL}(N)/\mathrm{SO}(N)$, with $\mathcal{V}$ in the fundamental representation, Δ reduces to the identity). By construction, $\mathcal{M}$ is invariant under H, while its transformation under G is given by

$$\delta\mathcal{M} = \Lambda\mathcal{M} + \mathcal{M}\Lambda^{\mathrm{T}}, \tag{4.26}$$

while the Lagrangian, as shown in equation (4.17), takes the form

$$\mathcal{L}_{\text{scalar}} = \frac{1}{8}\mathrm{Tr}(\partial_\mu \mathcal{M} \partial^\mu \mathcal{M}^{-1}). \tag{4.27}$$

In ungauged supergravity, scalars and vectors transform under a global symmetry group G in the following way:

$$\delta\mathcal{V} = \Lambda^\alpha t_\alpha \mathcal{V}, \tag{4.28}$$
$$\delta A_\mu^M = -\Lambda^\alpha (t_\alpha)^N{}_M A_\mu^N\ . \tag{4.29}$$

In addition, the n_v vector fields exhibit an Abelian gauge symmetry $\mathrm{U}(1)^{n_v}$:

$$\delta A_\mu^M = \partial_\mu \Lambda^M, \tag{4.30}$$

with $\Lambda^M = \Lambda^M(x)$ being the coordinate-dependent parameters. Higher-rank p-forms also possess corresponding Abelian tensor gauge symmetries.

4.2 | Embedding tensor mechanism

Gauging a subgroup $G_0 \subset G$, with G a global symmetry, consists of promoting G_0 to a local symmetry. These generators can be made local by introducing covariant derivatives. If we denote as t_α the generators of G and X_M the generators of G_0, then

$$\partial_\mu \to D_\mu \equiv \partial_\mu - g A_\mu^M X_M, \tag{4.31}$$

where g is the gauge coupling constant and $M = 1, \ldots, n_v$, with n_v the number of vector fields. Thus a general set of gauge generators, X_M, is parametrized as:

$$X_M \equiv \Theta_M{}^\alpha t_\alpha \in \mathfrak{g}, \tag{4.32}$$

where the term $\Theta_M{}^\alpha$ is the so-called embedding tensor. This object determines what is the linear combination of global symmetry generators hat assemble the gauge group G_0. For simplicity, $\Theta_M{}^\alpha$ can be seen as a constant $(n_v \times \dim G)$ matrix, with α and M as indices in the adjoint representations of G and the representation in which the vectors transform, respectively.

By parametrizing the gauge group generators with $\Theta_M{}^\alpha$, we maintain G-covariance throughout the construction. This approach allows us to express every single quantity related to the gauge group entirely in terms of the embedding tensor $\Theta_M{}^\alpha$. The gauge group G_0 is only specified when a particular $\Theta_M{}^\alpha$ is chosen, thereby breaking the global symmetry G.

Having introduced covariant derivatives, the theory must remain invariant under combined transformations:

$$\delta \mathcal{V} = g \Lambda^M X_M \mathcal{V}, \tag{4.33}$$

$$\delta A_\mu^M = D_\mu \Lambda^M \equiv \partial_\mu \Lambda^M + g A_\mu^N X_{NP}{}^M \Lambda^P, \tag{4.34}$$

where $\Lambda^M(x)$ is the local transformation parameter and $X_{NP}{}^M \equiv \Theta_N{}^\alpha (t_\alpha)_P{}^M$. However, not every $\Theta_M{}^\alpha$ ensures this covariance. To do so, the generators must close into a subalgebra of $\mathfrak{g}$, leading to non-trivial constraints on $\Theta_M{}^\alpha$.

The first constraint is quadratic in $\Theta_M{}^\alpha$ and ensures that the tensor is invariant under the action of the local gauge symmetry:

$$0 = \delta_{X_P} \Theta_M{}^\alpha = X_{PM}{}^N \Theta_N{}^\alpha - X_{P\beta}{}^\alpha \Theta_M{}^\beta = \Theta_P{}^\beta (t_\beta)_M{}^N \Theta_N{}^\alpha + \Theta_P{}^\beta \omega_{\beta\gamma}{}^\alpha \Theta_M{}^\gamma, \tag{4.35}$$

where $X_{P\beta}{}^\gamma = \Theta_P{}^\alpha (t_\alpha)_\beta{}^\gamma = -\Theta_P{}^\alpha \omega_{\alpha\beta}{}^\gamma$. This invariance implies that the generators form a closed algebra:

$$[X_M, X_N] = -X_{MN}{}^P X_P, \quad \text{with} \quad X_{MN}{}^P = \Theta_M{}^\alpha (t_\alpha)_N{}^P. \tag{4.36}$$

On the other hand, an additional linear constraint on $\Theta_M{}^\alpha$ is generally imposed by supersymmetry to ensure consistency with the deformed supergravity theory. The specific form of this constraint depends on the space-time dimensions and the number of supersymmetries.

The embedding tensor resides in the tensor product:

$$\Theta_M{}^\alpha : R_v^* \otimes R_{\text{adj}} = R_v^* \oplus \cdots \ , \tag{4.37}$$

where R_v^* denotes the dual representation to R_v, the representation in which the vector fields transform. The linear representation constraint takes the form:

$$\mathbb{P}\Theta = 0, \tag{4.38}$$

restricting Θ to certain representations on the right-hand side of the tensor product.

To summarize , the process of gauging a subgroup G_0 within the global symmetry group G a theory relies on the fact that the embedding tensor $\Theta_M{}^\alpha$ satisfies both quadratic and linear constraints. This guaranteeing the consistency and supersymmetry closure of the gauged supergravity theory.

4.2.1. Fermion shifts

Despite the introduction of covariant derivatives is crucial for the new gauge symmetry, it generates some tension with supersymmetry. To solve this problem it is necessary to consider additional terms in the action: a scalar potential, fermion masses, modified field strenghts,.... However, this is not sufficient: we actually need to extended the supersymmetric transformations of some fields. Let us note that, because these modifications are a consequence of the gauging procedure, they all depend on $\Theta_M{}^\alpha$ and, consequently, disappear when going back to the ungauged theory.

Let us consider the following schematic modifications of the ungauged Lagrangian:

$$\mathcal{L} = \mathcal{L}_0[\partial \to D, \mathcal{F}_0 \to \mathcal{H}] + \mathcal{L}_{\text{top}} + \mathcal{L}_{\text{ferm.mass}} + \mathcal{L}_{\text{pot}} \ . \tag{4.39}$$

Here $\mathcal{L}_0$ is the ungauged Lagrangian where every derivative must be substituted with a covariant derivative, and all p-form field strengths $\mathcal{F}_0$ must be replaced with the covariant ones $\mathcal{H}$, $\mathcal{L}_{\text{top}}$ is a topological term, $\mathcal{L}_{\text{ferm.mass.}}$ are all the bilinear couplings of fermions that do not involve p-form gauge fields or derivatives and $\mathcal{L}_{\text{pot}}$ is a scalar potential.

In the gauged theory, these couplings are required to cancel terms in the supersymmetry variations of the Lagrangian that arise due to the new gauge field couplings. However, this process is not enough to compensate all the new terms. We also need to change the variation of the fermions in this way

$$\delta_\epsilon \psi_\mu^a = \text{ungauged terms} + g A_{1b}{}^a \epsilon^b, \tag{4.40}$$

$$\delta_\epsilon \chi^m = \text{ungauged terms} + g A_{2a}{}^m \epsilon^a, \tag{4.41}$$

$$\delta_\epsilon \lambda^c = \text{ungauged terms} + g A_{2a}{}^c \epsilon^a, \tag{4.42}$$

where $\epsilon^a(x)$ is the parameter of supersymmetry transformations, and ψ_μ^a are the gravitini, χ^m are the matter fermions from the gravity multiplet and λ^c are the matter fermions from the vector multiplets. Here the indices a, m and c are related with some representation of the holonomy group. The holonomy group is expressed as $H = H_R \times H_{\text{matt}}$, λ^c acts as a vector under H_{matt}, and ψ^a_μ and χ^m act as a singlet under H_{matt}. A_1 and A_2 are tensors that generically depend on scalar fields and are linear on the embedding tensor. These tensors are called *fermion shifts*. The reason for this name is that, as we can see in (4.40),(4.41) and (4.42), they parametrize a shift with respect to the ungauged theory.

Substituting (4.40), (4.41) and (4.42) in $\mathcal{L}_{\text{ferm.mass}}$ when the variation is performed, we find terms of order g^2. To cancel these terms we need to introduce a scalar potential of the form

$$e^{-1}\mathcal{L}_{\text{pot}} = -g^2 V = 2g^2 \left(A^b_{1a}\bar{A}^a_{1b} - A^m_{2a}\bar{A}^a_{2m} - A^c_{2a}\bar{A}^a_{2c} \right), \tag{4.43}$$

where the bar means complex conjugate. Supersymmetry demans the following quadratic constraint on A_1 and A_2:

$$A^c_{1a}\bar{A}^b_{1c} - A^m_{2a}\bar{A}^b_{2m} - A^c_{2a}\bar{A}^b_{2c} = -\frac{1}{2r}\delta^b_a V, \tag{4.44}$$

where $r = \delta^a_a$ is the dimension of the gravitini representation. Let us note that, despite $\mathcal{L}_{\text{pot}}$ can have a highly non linear dependence on the scalar fields, it is quadratic in the embedding tensor.

4.2.2. Vacuum solutions

We usually assume that a vacuum state must be Lorentz and translation invariant. While the former requirement requires the gauge fields and fermions to vanish, the latter implies that the kinetic terms of the scalar fields must be discarded and the scalar fields can be constant. Consequently, the classical approximation to the vacuum state is obtained by finding extrema of the scalar potential $V(\phi)$. The values of the scalar fields at the critical points are usually called 'vacuum expectation values'.

The scalar potential $V(\phi)$ takes the role of the effective cosmological constant once the scalars sit at a critical point:

$$\partial_i V(\phi_0) = 0. \tag{4.45}$$

At such a point the energy momentum tensor reduces to a spacetime constant, so that Einstein equations are solved by a maximally symmetric metric with an effective cosmological constant $\Lambda_{\text{eff}} \propto V(\phi_0)$. The sign of $V(\phi_0)$ selects Minkowski ($V = 0$), Anti de Sitter (AdS) ($V < 0$) or de Sitter (dS) ($V > 0$) backgrounds.

If, for instance, we consider an AdS vacuum solution, the Riemann curvature tensor is given by:

$$R_{\mu\nu\rho\lambda} = -\frac{2}{\alpha^2} g_{\mu[\rho} g_{\nu]\lambda}, \tag{4.46}$$

where α is the AdS radius.

The radius α can be related to the scalar potential V through the relation:

$$\alpha^2 = -\frac{(D-1)(D-2)}{2g^2 V}, \tag{4.47}$$

where D is the dimension of spacetime and g is a coupling constant.

Therefore, the Riemann curvature tensor becomes:

$$R_{\mu\nu\rho\lambda} = \frac{4}{(D-1)(D-2)} g^2 V g_{\mu[\rho} g_{\nu]\lambda}. \tag{4.48}$$

For $d = 4$, this simplifies to:

$$R_{\mu\nu\rho\lambda} = \frac{2}{3} g^2 V g_{\mu[\rho} g_{\nu]\lambda}. \tag{4.49}$$

Finally, vacua preserving some supersymmetry satisfy the above conditions *and* the fermionic supersymmetry variations vanish for some nonzero Killing spinor(s) ϵ:

$$\delta\psi_\mu(\phi_0, F) = 0, \qquad \delta\chi^i(\phi_0, F) = 0,$$

typically leading to first–order differential/algebraic conditions on the scalars and fluxes (rather than just the second–order extremality conditions). Supersymmetric AdS vacua are the paradigmatic examples: their preserved supercharges often guarantee perturbative stability and provide extra control over the spectrum, and they frequently require nontrivial flux configurations rather than a completely vanishing p–form sector.

In supersymmetric theories, the amount of supersymmetry preserved by a given solution is determined by solving the so called Killing spinor equation, which originates from the supersymmetric transformations of the fermionic fields.

For a solution to be supersymmetric, both bosonic and fermionic supersymmetric variations must vanish. In general, the variation of the bosonic fields is proportional to the fermionic fields, while the variation of the fermionic fields is proportional to the bosonic fields. The proportionality factor involves a spinor parameter ϵ, which encodes the details of the supersymmetry transformation.

However, if we are interested in vacuum solutions, then the condition δbosons $= 0$ is trivially satisfied, as δbosons $\propto \bar{\epsilon}$ fermion. Thus, we only need to solve the equation δfermions $= 0$ in terms of the Killing spinor ϵ. This willdetermine the number preserved supersymmetries.

Additionally, by considering the Killing spinor equation and evaluating the fermionic variations in the vacuum, we obtain:

$$0 \stackrel{!}{=} \delta\psi^i_\mu = 2D_\mu\epsilon^i - \frac{2}{3} g A_1^{ij} \Gamma_\mu \epsilon_j, \tag{4.50}$$

$$0 \stackrel{!}{=} \delta\chi^i = -\frac{4}{3} i g A_2^{ji} \epsilon_j, \tag{4.51}$$

$$0 \stackrel{!}{=} \delta\lambda^i_a = 2igA_{2a}^{ji} \epsilon_j. \tag{4.52}$$

From the Killing spinor equation for the gravitini, we have:

$$D_\mu\epsilon^i = \frac{1}{3} g A_1^{ij} \Gamma_\mu \epsilon_j. \tag{4.53}$$

Let us compute the commutator[3]

$$\begin{aligned}
[D_\mu, D_\nu]\epsilon^i &= D_{[\mu|}\left(\frac{1}{3}gA_1^{ij}\Gamma_{|\nu]}\epsilon_j\right) \\
&= D_{[\mu|}\left(\frac{1}{3}gA_1^{ij}\Gamma_{|\nu]}B(\epsilon^j)^*\right) \\
&= \frac{1}{3}gA_1^{ij}\Gamma_{[\nu}B(D_{\mu]}\epsilon^j)^* \\
&= \frac{1}{9}g^2A_1^{ij}\Gamma_{[\nu|}B(A_1^{jk}\Gamma_{|\mu]}\epsilon_k)^* \\
&= \frac{1}{9}g^2\Gamma_{[\nu|}B\Gamma^*_{|\mu]}A_1^{ij}({A_1}^{jk}\epsilon_k)^* \\
&= -\frac{1}{9}g^2\Gamma_{[\mu}B\Gamma^*_{\nu]}A_1^{ij}({A_1}^{jk}\epsilon_k)^*,
\end{aligned}$$

On the other hand

$$\begin{aligned}
[D_\mu, D_\nu]\,\epsilon^i &= -\frac{1}{4}R_{\mu\nu}{}^{\rho\lambda}\Gamma_{\rho\lambda}\epsilon^i \\
&= \frac{1}{4}R_{\mu\nu}{}^{\rho\lambda}\Gamma_{[\rho}B\Gamma^*_{\lambda]}B^*\epsilon^i \\
&= \frac{1}{4}g^{\rho\sigma}g^{\lambda\tau}R_{\mu\nu\sigma\tau}\Gamma_{[\rho}B\Gamma^*_{\lambda]}B^*\epsilon^i \\
&= \frac{1}{4}g^{\rho\sigma}g^{\lambda\tau}\frac{4}{6}g^2Vg_{\mu[\sigma}g_{\nu]\tau}\Gamma_{[\rho}B\Gamma^*_{\lambda]}B^*\epsilon^i \\
&= \frac{1}{12}(\delta^\rho_\mu\delta^\lambda_\nu - g_{\mu\nu}g^{\rho\lambda})g^2V\Gamma_{[\rho}B\Gamma^*_{\lambda]}B^*\epsilon^i \\
&= \frac{1}{12}g^2V\Gamma_{[\mu}B\Gamma^*_{\nu]}B^*\epsilon^i.
\end{aligned}$$

Therefore

$$\Gamma_{[\mu}B\Gamma^*_{\nu]}A_1^{ij}({A_1}^{jk}\epsilon_k)^* = -\frac{3}{4}V\Gamma_{[\mu}B\Gamma^*_{\nu]}B^*\epsilon^i. \tag{4.54}$$

If we take ${A_1}^{ij}\epsilon_j = \sqrt{-\frac{3}{4}V}\epsilon^i$, we can check that (4.54) is satisfied:

$$\begin{aligned}
\Gamma_{[\mu}B\Gamma^*_{\nu]}A_1^{ij}({A_1}^{jk}\epsilon_k)^* &= \sqrt{-\frac{3}{4}V}\Gamma_{[\mu}B\Gamma^*_{\nu]}B^*A_1^{ij}B(\epsilon^j)^* \quad (4.55) \\
&= \sqrt{-\frac{3}{4}V}\Gamma_{[mu}B\Gamma^*_{\nu]}A_1^{ij}B(\epsilon^j)^* \\
&= \sqrt{-\frac{3}{4}V}\Gamma_{[\mu}B\Gamma^*_{\nu]}B^*A_1^{ij}\epsilon_j \quad (4.56) \\
&= -\frac{3}{4}V\Gamma_{[\mu}B\Gamma^*_{\nu]}B^*\epsilon^i. \quad (4.57)
\end{aligned}$$

[3] In the chiral representation of the Gamma-matrices, we define $B = i\Gamma_2\Gamma_5 = \begin{pmatrix} 0 & \epsilon \\ -\epsilon & 0 \end{pmatrix}$, with ϵ being the two-dimensional Levi-Civita symbol. Consequently, we have $B = B^* = B^{-1}$. Moreover, we use the relation $\epsilon_i = B(\epsilon^i)^*$.

Thus, from (4.51) and (4.52) we conclude that

$$A_1{}^{ij}\epsilon_j = \sqrt{-\frac{3}{4}V}\epsilon^i, \qquad A_2^{ji}\epsilon_j = 0, \qquad A_{2a}^{ji}\epsilon_j = 0. \tag{4.58}$$

That is to say, these are the required conditions on ϵ^i for a vacuum solution to preserve some supersymmetry.

4.3 | Scalar potential

The explicit form of the scalar potential in a gauge supergravity theory is determined by the choice of gauge fields and their couplings. Generally, the scalar potential $V(\phi)$ can be expressed as a non linear function of the scalar fields ϕ which is quadratic in the embedding tensor. Since the scalar potential is quadratic in the fermion shift matrices, and these matrices depend linearly on the embedding tensor, the potential can always be written in the form

$$V = g^2 V_{\alpha\beta}^{MN}\Theta_M^\alpha\Theta_N^\beta, \tag{4.59}$$

where the potential is written in terms of the embedding tensor and a scalar-dependent matrix $V_{\alpha\beta}^{MN}$. Let us review some examples.

In the case of $\mathcal{N}=8$ $D=4$ supergravity, the scalar potential can be equivalently written as [122]

$$V = g^2\left(X_{MN}{}^R X_{PQ}{}^S \mathcal{M}^{MP}\mathcal{M}^{NQ}\mathcal{M}_{RS} + 7X_{MN}{}^Q X_{PQ}{}^N \mathcal{M}^{MP}\right), \tag{4.60}$$

with $X_{MN}{}^K$ defined in equation (4.36) as a function of the embedding tensor, and the positive definite scalar matrices $\mathcal{M}_{MN}$ defined in (4.25).

In $\mathcal{N}=1$ $D=4$ supergravity, the scalar potential is given by:

$$V = e^K\left(K^{i\bar{j}}D_iWD_{\bar{j}}\bar{W} - 3|W|^2\right), \tag{4.61}$$

where W is the superpotential, K is the Kähler potential, K^{ij} is the inverse Kähler metric, depending on the geometry of the moduli space of the scalar fields, and $D_iW = \partial_iW + (\partial_iK)W$ denotes the Kähler covariant derivative of the superpotential.

4.3.1. Going to the origin

Let us explore the procedure introduced in [102] to find extrema of the scalar potential $V(\phi)$. As we will see in Chapter 6, this method turns out to be pretty useful for a systematic exploration of vacua. That is to say, it is an efficient method for identifying extrema of the scalar potential $V(\phi)$.

As we have mentioned earlier, the scalar potential depends on scalar fields ϕ, represented by coset elements $L(\phi) \in G/H$, and an embedding tensor Θ:

$$V(\phi) = V\left(L(\phi), \Theta\right). \tag{4.62}$$

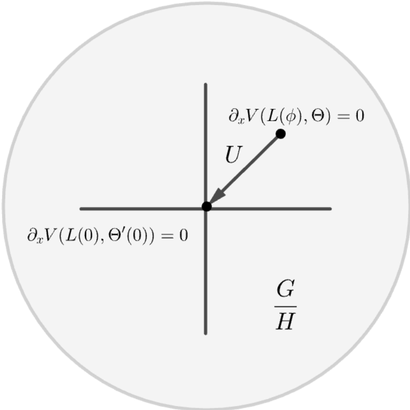

Figure 4.1: Going to the origin

Due to the non linear functional dependence of the scalar fields in V, solving these equations directly can be difficult. Typically, one simplifies the problem by considering symmetry-invariant subsets of scalars (consistent truncations) or using numerical techniques.

A more elegant alternative exploits the geometric nature of the scalar manifold, a homogeneous coset space G/H. Any scalar configuration can be related to the origin through transformations in the global symmetry group G. The scalar potential remains invariant under simultaneous transformations of the scalar fields and the embedding tensor, allowing it to be expressed only in terms of $L^{-1}(\phi)\Theta$:

$$V(\phi) = V\left(0, L^{-1}(\phi)\Theta\right). \tag{4.63}$$

This implies that any critical point can be mapped to the origin ($\phi = 0$), where the scalar potential and its derivatives simplify to quadratic expressions in Θ. Thus, finding vacua reduces to solving quadratic algebraic equations involving only the embedding tensor.

Let us see this in more detail. Consider a transformation $U \in G$ acting linearly on the coset representatives:

$$UL(\phi) = L(\phi')h(\phi, \phi'), \quad h \in H, \tag{4.64}$$

with a simultaneous transformation of the embedding tensor:

$$\Theta \to \Theta' = U\Theta. \tag{4.65}$$

Since the scalar potential involves only H-invariant quantities, these transformations imply:

$$V(L(\phi), \Theta') = V(L(\phi'), \Theta). \tag{4.66}$$

Therefore, this expression enables us to explore the homogeneous scalar manifold by evaluating the potential at the reference point and systematically varying Θ. The invariance of this point under the compact subgroup H ensures variations of Θ along non-compact directions of G corresponding to distinct scalar configurations.

This process of finding extrema of the scalar potential is called ”going to the origin”.

4.4 | Higher dimensional origin

Some gauged supergravities in lower dimensions can be obtained from compactifications of higher dimensional theories [4]. In this subsection we study compactifications on group manifolds for a purely gravitational theory as a toy model and for type II theories. For further types of compactifications we refer to [121, 120].

4.4.1. No twist no party: compactification on group manifolds

We will consider a D-dimensional gravitational theory compactified to d dimensions, with the spacetime coordinates split as $x^{\mathcal{M}} = (x^\mu, y^m)$, where $\mathcal{M} = 0, 1, \ldots, D-1$, $\mu = 0, 1, \ldots, d$, and $m = d+1, \ldots, D-1$. The internal compact space is taken to be a group manifold, as such geometries naturally induce a non-trivial scalar potential upon compactification, which can contribute to the stabilization of the moduli. We will begin with the definition of group manifold.

A *group manifold* is a differentiable manifold whose points can be identified with the elements of a Lie group G. If we consider coordinates y^m, then each group element is written as $g(y^m)$, and the dimension of the manifold, D, is equal to the dimension of the group. The structure of the group naturally induces coordinate transformations on the manifold through group multiplication. More precisely, left and right multiplication by a constant group element, *i.e.*, $g \mapsto \Lambda_L g$ or $g \mapsto g\Lambda_R$, define maps that move points along the manifold. These transformations act transitively, meaning that any point can be reached from any other one by applying them.

However, these transformations are not in general isometries of the metric on the manifold. To guarantee that left multiplication defines an isometry, one can introduce a special type of metric. We define

$$ds_G^2 = g_{mn}\,\sigma^m\sigma^n, \qquad T_m\sigma^m = g^{-1}dg, \tag{4.67}$$

where g_{mn} is an arbitrary metric, T_m are the generators of the Lie algebra associated with G, $g = g(y^m)$ are elements of the group, and σ^m is a set of one-forms known as Maurer–Cartan forms. In terms of the coordinates y^p, one can write $\sigma^m = \sigma^m(y^p) = U^m{}_n(y^p)\,dy^n$, for some matrix $U^m{}_n$ depending on the point of the manifold.

Since the Maurer–Cartan forms are invariant under left multiplication, the metric defined above is also invariant under such transformations. For this reason, it is referred to as a *left-invariant metric*. A manifold equipped with this type of metric possesses a transitive group of isometries generated by left multiplication. The corresponding infinitesimal generators are Killing vector fields L_n, which satisfy the Maurer-Cartan equations:

$$[L_m, L_n] = \omega_{mn}{}^p\,L_p, \tag{4.68}$$

[4]See [119] for a systematic analysis of geometric and non-geometric gaugings in (half-)maximal supergravities for $D \geq 7$.

where the structure constants ${\omega_{mn}}^p$ are given by

$$
{\omega_{mn}}^p = -2{(U^{-1})^r}_m \, {(U^{-1})^s}_n \, \partial_r({U^p}_s). \tag{4.69}
$$

These constants are the structure constants of the group G and are independent of the coordinates y^m, as guaranteed by Lie's second theorem.

It is worth noting that while left multiplication defines isometries of the metric (4.67), right multiplication does not, in general, preserve it. Nevertheless, if the components g_{mn} are chosen to coincide with the Cartan–Killing metric of the group G, then the metric becomes invariant under both left and right group actions. In this case, the full isometry group is given by $G_L \times G_R$, where G_L and G_R denote the actions of the group from the left and the right, respectively.

To understand how these group manifolds appear in dimensional reductions, one can start by considering a toroidal reduction Ansatz. In such case, the higher-dimensional metric can be decomposed as

$$
\hat{ds}^2 = e^{2\alpha\Phi} ds^2 + e^{2\beta\Phi} M_{mn} \left(dy^m + A^m_\mu dx^\mu\right)\left(dy^n + A^n_\mu dx^\mu\right), \tag{4.70}
$$

where α and β are defined as:

$$
\alpha^2 = \frac{D-d}{2(D-2)(d-2)}, \qquad \beta = -\frac{(d-2)\alpha}{D-d}, \tag{4.71}
$$

and M_{mn} encodes the moduli of the internal space.

One can introduce a local transformation matrix ${U^m}_n(y^p)$ and reinterpret the toroidal Ansatz

$$
\begin{aligned}
\hat{ds}^2 &= e^{2\alpha\Phi} ds^2 + e^{2\beta\Phi} {U^m}_p {U^q}_n M_{pq} \left(dy^m + {(U^{-1})^m}_r A^r_\mu dx^\mu\right)\left(dy^n + {(U^{-1})^n}_s A^s_\mu dx^\mu\right) \\
&= e^{2\alpha\Phi} ds^2 + e^{2\beta\Phi} M_{mn} (\sigma^m + A^m_\mu dx^\mu)(\sigma^n + A^n_\mu dx^\mu),
\end{aligned} \tag{4.72}
$$

where the new one-forms are defined as $\sigma^m = {U^m}_n dy^n$.

The internal part of the metric now becomes

$$
ds^2_G = e^{2\beta\Phi} M_{mn} \sigma^m \sigma^n, \tag{4.73}
$$

which corresponds precisely to a left-invariant metric on a group manifold.

The reduction of the higher-dimensional Einstein–Hilbert term then yields the following Lagrangian:

$$
\mathcal{L} = \sqrt{-g}\left[R + \tfrac{1}{4}\mathrm{Tr}(D\mathcal{M}D\mathcal{M}^{-1}) - \tfrac{1}{2}(\partial\Phi)^2 - \tfrac{1}{4}e^{2(\alpha-\beta)\Phi} F^m_{\mu\nu}\mathcal{M}_{mn}F^{n\,\mu\nu} - V\right], \tag{4.74}
$$

with field strengths and covariant derivatives defined by

$$
F^m = 2\partial A^m - {\omega_{np}}^m A^n \wedge A^p, \tag{4.75}
$$

$$
DM_{mn} = \partial M_{mn} + 2{\omega_{q(m}}^p A^q M_{n)p}. \tag{4.76}
$$

In addition to the kinetic terms, a scalar potential emerges in the reduced theory:

$$V = \tfrac{1}{4}e^{2(\beta-\alpha)\Phi}\left[2\mathcal{M}^{mq}\omega_{mp}{}^{r}\omega_{qr}{}^{p} + \mathcal{M}^{mq}\mathcal{M}^{nr}M_{ps}\omega_{mn}{}^{p}\omega_{qr}{}^{s}\right]. \tag{4.77}$$

The field strengths are deformed by the presence of structure constants, and the scalar sector acquires a nontrivial potential. These deformations reflect the non-Abelian nature of the internal space, and depend linearly and quadratically on the structure constants, respectively.

4.4.2. Bulk compactification of type II theories

The pseudo-action for type II supergravities in the string frame can be expressed as:

$$S_{\text{II}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(e^{-2\Phi}\left(\mathcal{R} + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2\right) - \frac{1}{2}\sum_p \frac{|F_{(p)}|^2}{p!}\right) + S_{WZ}\ , \tag{4.78}$$

where $p = 0, 2, 4$ corresponds to massive type IIA, and $p = 1, 3, 5$ applies to type IIB theory. Here, S_{WZ} represents a topological term, with different explicit forms for the IIA and IIB cases.

In order to carry out the dimensional reduction of type II down to d dimensions, we parametrize the ten-dimensional metric $g_{\mathcal{M}\mathcal{N}}$ in terms of the d-dimensional one and the moduli describing the $(10-d)$-dimensional internal metric. In particular, by picking

$$ds^2_{(10)} = g_{\mathcal{M}\mathcal{N}}\,dx^{\mathcal{M}}\otimes dx^{\mathcal{N}} = \tau^{-2}\,g^{(d)}_{\mu\nu}\,dx^{\mu}\otimes dx^{\nu} + \rho\,ds^2_{(10-d)}\ , \tag{4.79}$$

the universal moduli ρ and τ are singled out. The rest of the moduli, which describe volume-preserving deformations of the internal geometry, are contained within $ds^2_{(10-d)}$. In addition to that, we introduce local indices m, n as follows:

$$ds^2_{(10-d)} = \mathcal{M}_{mn}\,v^m\otimes v^n\ , \tag{4.80}$$

where $\mathcal{M}_{mn}$ parametrizes the coset $\text{SL}(10-d,\mathbb{R})/\text{SO}(10-d)$ with $\det\mathcal{M} = 1$.

To obtain the d-dimensional gravity action in the Einstein frame after compactification, we impose the constraint:

$$\rho^{5-d/2} \overset{!}{=} e^{2\Phi}\,\tau^{d-2}\ , \tag{4.81}$$

which implies that ρ and τ fix the internal volume and the string coupling.

Now, let's break down the type II effective action. The determinant of the metric reduces as follows:

$$\sqrt{-g} = \tau^{-d}\,\rho^{5-d/2}\,\sqrt{-g_{(d)}}\sqrt{g_{(10-d)}}\ . \tag{4.82}$$

Reducing the Einstein term in the original action (4.78) yields[5]:

[5] Note that $\mathcal{R}^{(10)} = \tau^2\,\mathcal{R}^{(d)} + \rho^{-1}\,\mathcal{R}^{(10-d)}$.

$$\int d^{10}x\,\sqrt{-g}\,e^{-2\Phi}\mathcal{R}^{(10)} = \int d^dx\,\sqrt{-g_{(d)}}\,\left(\tau^{2-d}\,\rho^{5-d/2}\,e^{-2\Phi}\,\mathcal{R}^{(d)} + \tau^{-d}\,\rho^{4-d/2}\,e^{-2\Phi}\,\mathcal{R}^{(10-d)}\right)$$
$$= \int d^dx\,\sqrt{-g_{(d)}}\,\left(\mathcal{R}^{(d)} - V_\omega\right)\ , \qquad (4.83)$$

where $V_\omega \equiv -\rho^{-1}\,\tau^{-2}\,\mathcal{R}^{(10-d)}$. In case of twisted toroidal compactifications, $\mathcal{R}^{(10-d)}$ is giving by:

$$\mathcal{R}^{(10-d)} = -\frac{1}{4}\,\mathcal{M}_{mq}\mathcal{M}^{nr}\mathcal{M}^{ps}\,\omega_{np}{}^{q}\,\omega_{rs}{}^{m} - \frac{1}{2}\,\mathcal{M}^{np}\,\omega_{mn}{}^{q}\,\omega_{qp}{}^{m}\ , \qquad (4.84)$$

where $\mathcal{M}^{mn}$ is the inverse of $\mathcal{M}_{mn}$ and $\omega_{mn}{}^{p}$ are the structure constants entering the Maurer-Cartan equation

$$dv^m + \frac{1}{2}\omega_{np}{}^{m}\,v^n \wedge v^p = 0\ . \qquad (4.85)$$

This, in turn, implies the Jacobi identities as an integrability condition,

$$\omega_{[mn}{}^{r}\,\omega_{p]r}{}^{q} = 0\ . \qquad (4.86)$$

In addition to this, we will ask the structure constants to fulfill the unimodularity condition $\omega_{mn}{}^{n} = 0$ for consistency, as we are performing the compactification at the level of the action.

Next, considering the scalar potential from the $H_{(3)}$ flux. Reducing the corresponding term of the action (4.78) yields

$$\int d^{10}x\,\sqrt{-g}\left(-\frac{1}{12}\,e^{-2\Phi}\,|H_{(3)}|^2\right) = \int d^dx\,\sqrt{-g_{(d)}}\left(-\frac{1}{12}\,H_{mnp}\,H^{mnp}\,\rho^{-3}\,\tau^{-2}\right)\ , \qquad (4.87)$$

giving a contribution of $V_H \equiv \frac{1}{12}\,H_{mnp}\,H^{mnp}\,\rho^{-3}\,\tau^{-2}$.

For the R-R p-forms, their contribution to the scalar potential is:

$$\int d^{10}x\,\sqrt{-g}\left(-\frac{1}{2p!}\,|F_{(p)}|^2\right) = \int d^dx\,\sqrt{-g_{(d)}}\left(-\frac{1}{2p!}\,F_{m_1\cdots m_p}\,F^{m_1\cdots m_p}\,\rho^{5-p-d/2}\,\tau^{-6}\right)\ , \qquad (4.88)$$

wich results in $V_{F_p} = \frac{1}{2p!}\,F_{m_1\cdots m_p}\,F^{m_1\cdots m_p}\,\rho^{5-p-d/2}\,\tau^{-6}$.

Finally, as the 10-dimensional Chern-Simons term does not contribute to the potential, the reduced d-dimensional theory is given by the following action:

$$S_d = \int d^dx\sqrt{-g_{(d)}}\,\left(\mathcal{R}^{(d)} + 2\mathcal{L}_{\text{kin}} - V\right)\ , \qquad (4.89)$$

where the full scalar potential arising from the bulk is composed of:

$$V = V_\omega + V_H + \sum_p V_{F_p}\ . \qquad (4.90)$$

The kinetic term for the moduli, which span a $\mathbb{R}^+_\rho \times \mathbb{R}^+_\tau \times \mathrm{SL}(10-d,\mathbb{R})/\mathrm{SO}(10-d)$ geometry, is given by

$$\mathcal{L}_{\text{kin}} = -\left(\frac{10-d}{8}\right)^2 \frac{(\partial\rho)^2}{\rho^2} - \left(\frac{2-d}{4}\right)^2 \frac{(\partial\tau)^2}{\tau^2} + \frac{1}{8}\text{Tr}\left(\partial\mathcal{M}\partial\mathcal{M}^{-1}\right) \ . \tag{4.91}$$

4.4.3. Tadpoles and Green-Schwarz mechanism

In type II theories, the Bianchi identities for the Ramond–Ramond fluxes are modified in the presence of localized sources such as D-branes and orientifold planes. To treat these fluxes in a unified way, it is useful to employ the democratic formulation [77], where all RR potentials are included as part of a formal sum. The RR field strengths are given by

$$\mathbf{F} = d\mathbf{C} + m\, e^{B_{(2)}} - H_{(3)} \wedge \mathbf{C}, \tag{4.92}$$

where

$$\mathbf{F} = \sum_{n=0,\,1/2}^{5,\,9/2} F_{(2n)}, \qquad \mathbf{C} = \sum_{n=1,\,1/2}^{5,\,9/2} C_{(2n-1)}. \tag{4.93}$$

and $m = F_{(0)}$ in massive type IIA. These field strengths satisfy a duality condition given by

$$F_{(n)} = (-1)^{\lfloor n/2 \rfloor} \star F_{(10-n)}, \tag{4.94}$$

where $\star$ denotes the Hodge dual in ten dimensions and $\lfloor n/2 \rfloor$ is the floor function applied to $n/2$. The Bianchi identities for the RR field strengths then are

$$d\mathbf{F} - H_{(3)} \wedge \mathbf{F} = 0. \tag{4.95}$$

When localized sources are included, the Bianchi identities are modified [64] as

$$dF_{(n)} = H_{(3)} \wedge F_{(n-2)} + (2\pi\sqrt{\alpha'})^{n-1} \rho^{\text{loc}}_{8-n}, \tag{4.96}$$

where ρ^{loc}_{8-n} denotes the dimensionless charge density of the $8-n$-dimensional (in space only) magnetic source for $F_{(n)}$, which contains a $\delta^{n+1}(\vec{x} - \vec{x}_i)$.

Integrating the Bianchi identity over a compact cycle leads to a global consistency condition, known as the *tadpole cancellation condition.* This condition ensures that the total RR charge (from fluxes and sources) vanishes on compact internal manifolds.

Example (Type IIA): For D6-branes and O6-planes extended along spacetime and wrapped on a three-cycle $\tilde{\Sigma}_3$, integration of the Bianchi identity for $F_{(2)}$ over the dual cycle Σ_3 yields the condition

$$N_{\text{D6}}(\tilde{\Sigma}_3) - 2N_{\text{O6}}(\tilde{\Sigma}_3) + \frac{F_{(0)}}{2\pi\sqrt{\alpha'}} \int_{\Sigma_3} H_{(3)} = 0, \tag{4.97}$$

where $N_{\text{D6}}(\tilde{\Sigma}_3)$ and $N_{\text{O6}}(\tilde{\Sigma}_3)$ are the number of D6-branes and O6-planes wrapped on $\tilde{\Sigma}_3$, respectively.

Example (Type IIB): Similarly, in type IIB, D5-branes and O5-planes wrapped on a two-cycle $\tilde{\Sigma}_2$ act as sources for $F_{(3)}$. Integrating the Bianchi identity over the dual four-cycle Σ_4 gives

$$N_{\text{D5}}(\tilde{\Sigma}_2) - N_{\text{O5}}(\tilde{\Sigma}_2) + \frac{1}{(2\pi)^2\alpha'}\int_{\Sigma_4} H_{(3)} \wedge F_{(1)} = 0. \tag{4.98}$$

We can connect the tadpole cancellation conditions with the original Green–Schwarz mechanism in heterotic string theory.

In the heterotic case, the bosonic part of the ten-dimensional effective action contains the term

$$S_{\text{bos}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-g}\,e^{-2\Phi}\left(\mathcal{R} + 4\partial_{\mathcal{M}}\Phi\,\partial^{\mathcal{M}}\Phi - \frac{1}{2\cdot 3!}|\tilde{H}_{(3)}|^2 - \frac{\alpha'}{2\cdot 2!}\text{Tr}_v\,|\mathcal{F}_{(2)}|^2\right). \tag{4.99}$$

The crucial point is that the field strength of the NSNS two-form is not simply $dB_{(2)}$, but is modified by gauge and gravitational Chern–Simons terms:

$$\tilde{H}_{(3)} = dB_{(2)} - \frac{\alpha'}{4}\left(\omega_3 - \omega_3^{\text{grav}}\right), \tag{4.100}$$

where

$$\omega_3 = \text{Tr}_v\left(\mathcal{A}\wedge d\mathcal{A} - \frac{2i}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right), \qquad d\omega_3 = \text{Tr}_v\mathcal{F}\wedge\mathcal{F}, \tag{4.101}$$

and

$$d\omega_3^{\text{grav}} = \text{Tr}\,\mathcal{R}_{(2)}\wedge\mathcal{R}_{(2)}, \tag{4.102}$$

where $\mathcal{R}_{(2)}$ is the curvature 2-form.

Therefore the Bianchi identity for $\tilde{H}_{(3)}$ is modified to

$$d\tilde{H}_{(3)} = \frac{\alpha'}{4}\left(\text{Tr}\,\mathcal{R}_{(2)}\wedge\mathcal{R}_{(2)} - \text{Tr}_v\mathcal{F}\wedge\mathcal{F}\right). \tag{4.103}$$

This modification is directly related to anomaly cancellation. Ten-dimensional heterotic string theories are chiral and therefore may suffer from gauge, gravitational and mixed anomalies from one-loop hexagon diagrams. The potentially anomalous one-loop hexagon diagram gives a contribution whose relevant factorized structure is schematically

$$A_{\text{hex}} \simeq \text{Tr}_v\,\mathcal{F}^4\wedge\left(\text{Tr}\,\mathcal{R}_{(2)}^2 - \text{Tr}_v\,\mathcal{F}^2\right). \tag{4.104}$$

The Green–Schwarz mechanism cancels this anomaly through the exchange of the Kalb–Ramond field. In ten dimensions, this degree of freedom can be described electrically by the two-form potential $B_{(2)}$, or magnetically by its dual six-form potential $B_{(6)}$. Inserting the modified definition of $\tilde{H}_{(3)}$ into its kinetic term produces a cross coupling between $dB_{(2)}$ and the gauge and gravitational Chern–Simons three-forms. After dualizing $B_{(2)}$ to $B_{(6)}$ and integrating by parts, this coupling can be written schematically as

$$\int_{10d} B_{(6)} \wedge \left(\operatorname{Tr} \mathcal{R}_{(2)}^2 - \operatorname{Tr}_v \mathcal{F}^2\right). \tag{4.105}$$

On the other hand, there is also a one-loop correction coupling the two-form to four gauge field strengths,

$$S_{B_2 \mathcal{F}^4} = \int_{10d} B_{(2)} \wedge \operatorname{Tr}_v \mathcal{F}^4. \tag{4.106}$$

The combination of these two contributions has precisely the same factorized structure as the hexagon anomaly, and cancels it.

Thus, in the heterotic theory, anomaly cancellation is encoded geometrically in the modified Bianchi identity

$$d\tilde{H}_{(3)} = \frac{\alpha'}{4} \left(\operatorname{Tr} \mathcal{R}_{(2)} \wedge \mathcal{R}_{(2)} - \operatorname{Tr}_v \mathcal{F} \wedge \mathcal{F}\right). \tag{4.107}$$

Upon compactification on a compact internal manifold, the modified Bianchi identity gives a global consistency condition, in close analogy with the RR tadpole conditions discussed above. The point is that the right-hand side of the Bianchi identity acts as a magnetic source for $\tilde{H}_{(3)}$. If there are no localized NS5-brane sources, then $\tilde{H}_{(3)}$ is smooth on any closed internal four-cycle Σ_4, and Stokes' theorem gives

$$\int_{\Sigma_4} d\tilde{H}_{(3)} = \int_{\partial \Sigma_4} \tilde{H}_{(3)} = 0. \tag{4.108}$$

Integrating the Bianchi identity over Σ_4 therefore gives

$$\int_{\Sigma_4} \operatorname{Tr} \mathcal{R}_{(2)} \wedge \mathcal{R}_{(2)} = \int_{\Sigma_4} \operatorname{Tr}_v \mathcal{F} \wedge \mathcal{F}. \tag{4.109}$$

Thus the gravitational and gauge contributions to the magnetic charge of $\tilde{H}_{(3)}$ must cancel globally. This is the heterotic analogue of a tadpole cancellation condition.

Chapter 5

New IIB intersecting brane solutions

This chapter marks the beginning of the original results of this thesis. We study genuine supersymmetric brane intersections in type IIB string theory that preserve $(1+1)$-dimensional Lorentz symmetry. The full supergravity solutions are obtained in explicit analytic form, and their main physical properties are analyzed. The Ansatz for the spacetime dependence of the brane warp factors goes beyond the harmonic superposition principle. From the associated near-horizon geometries, we construct new families of AdS_3 vacua in type IIB and relate them to existing classifications in the literature. Finally, we discuss their holographic properties.

5.1 | D3 – D5 – D7 brane systems

In [22], deformations of $\mathcal{N}=4$ SYM_4 with partially SUSY preserving spacetime dependent profiles for the SYM couplings and the mass terms were studied. In particular, the field theory enjoys a rigid $\text{ISO}(1,1)\times\text{SO}(3)\times\text{SO}(3)$ symmetry and the deformation parameters generically depend on two coordinates. It was argued that their stringy origin is the presence of *defect branes* in the background [23].

On the supergravity side, an $\text{AdS}_5\times S^5$ geometry emerges when taking the near-horizon limit of the D3 brane metric. By placing D5 and D7 branes within the aforementioned as depicted in table 5.1 preserve four real supercharges, *i.e.* it is $\frac{1}{8}$-BPS, $(1+3)$D conformal symmetry is broken and one expects a lower-dimensional field theory description to emerge. Even though no lower-dimensional CFT is expected to emerge in the (new) near-horizon limit, we look for the supergravity description of the field theory construction illustrated in [22].

object	t	y	x_1	x_2	r	ϕ_1	ϕ_2	ρ	θ_1	θ_2
D3	×	×	×	×	∼	∼	∼	−	−	−
D5	×	×	×	−	×	×	×	∼	∼	∼
D7	×	×	−	−	×	×	×	×	×	×

Table 5.1: The $\frac{1}{8}$ BPS brane system underlying the intersection of D5 – D7 branes intersecting D3 branes. The symbols are × for directions the brane spans, ∼ for smeared (homogeneously distributed) directions, and − for directions the brane does not extend into.

When we analyze the Killing spinor projections for the D3, D5, and D7 branes, we require that

$$\Pi(\mathcal{O}_{\text{D3}})\,\epsilon \overset{!}{=} \Pi(\mathcal{O}_{\text{D5}})\,\epsilon \overset{!}{=} \Pi(\mathcal{O}_{\text{D7}})\,\epsilon \overset{!}{=} \epsilon\ . \tag{5.1}$$

This condition ensures that four real supercharges are preserved because admits a space of solutions parametrized by four independent real parameters, thus giving rise to a $\frac{1}{8}$ BPS brane intersection.

The 10D supergravity Ansatz we consider is[1]

$$\begin{aligned} ds^2_{10} =& H_{\rm D3}^{-1/2} H_{\rm D5}^{-1/2} H_{\rm D7}^{-1/2} ds^2_{\rm Mkw_2} + H_{\rm D3}^{-1/2} H_{\rm D7}^{1/2} \left(H_{\rm D5}^{-1/2} dx_1^2 + H_{\rm D5}^{1/2} dx_2^2 \right) \\ &+ H_{\rm D3}^{1/2} H_{\rm D5}^{-1/2} H_{\rm D7}^{-1/2} \left(dr^2 + r^2 ds^2_{S^2} \right) + H_{\rm D3}^{1/2} H_{\rm D5}^{1/2} H_{\rm D7}^{-1/2} \left(d\rho^2 + \rho^2 ds^2_{\tilde{S}^2} \right) \ , \end{aligned} \tag{5.2}$$

$$C_{(4)} = H_{\rm D7} H_{\rm D3}^{-1} \, \text{vol}_{\rm Mkw_2} \wedge dx_1 \wedge dx_2 |_{\rm SD} \ , \tag{5.3}$$

$$C_{(6)} = H_{\rm D5}^{-1} \, \text{vol}_{\rm Mkw_2} \wedge dx_1 \wedge \text{vol}_{\mathbb{R}^3} \ , \tag{5.4}$$

$$C_{(8)} = H_{\rm D3} H_{\rm D7}^{-1} \, \text{vol}_{\rm Mkw_2} \wedge \text{vol}_{\mathbb{R}^3} \wedge \text{vol}_{\tilde{\mathbb{R}}^3} \ , \tag{5.5}$$

$$e^{\Phi} = H_{\rm D5}^{-1/2} H_{\rm D7}^{-1} \ , \tag{5.6}$$

where $ds^2_{\rm Mkw_2} \equiv (-dt^2 + dy^2)$, $ds^2_{S^2} \equiv (d\phi_1^2 + \sin^2\phi_1 d\phi_2^2)$, $ds^2_{\tilde{S}^2} \equiv (d\theta_1^2 + \sin^2\theta_1 d\theta_2^2)$, and $(\cdots)|_{\rm SD}$ denotes projection onto the self-dual field. The warp factors appearing in the above Ansatz are respectively assumed to have the following spacetime dependence: $H_{\rm D3} = H_{\rm D3}(\rho)$, $H_{\rm D5} = H_{\rm D5}(x_2)$ and $H_{\rm D7} = H_{\rm D7}(x_1, x_2)$.

The complete set of Bianchi identities (2.54) and equations of motion (2.55) – (2.58) is satisfied upon imposing the following differential equations

$$\partial^2_{x_2} H_{\rm D5} \overset{!}{=} 0 \ , \tag{5.7}$$

$$\Delta_{\tilde{\mathbb{R}}^3} H_{\rm D3} \ \equiv \ \partial^2_{\rho} H_{\rm D3} + \frac{2}{\rho} \partial_{\rho} H_{\rm D3} \overset{!}{=} 0 \ , \tag{5.8}$$

$$H_{\rm D5} \, \partial^2_{x_1} H_{\rm D7} + \partial^2_{x_2} H_{\rm D7} \overset{!}{=} 0 \ . \tag{5.9}$$

It may be worth mentioning that, while the first two differential conditions impose harmonicity and can be integrated in fully generality, the third one admits *non-harmonic* solutions for $H_{\rm D7}$, and in particular its explicit form crucially depends on the choice of $H_{\rm D5}$.

Equation (5.7) is solved by

$$H_{\rm D5} \ = \ h_{\rm D5} \ + \ Q_{\rm D5} x_2 \ , \tag{5.10}$$

where $h_{\rm D5}$ and $Q_{\rm D5}$ are some real integration constants, while (5.8) yields

$$H_{\rm D3} \ = \ h_{\rm D3} \ + \ \frac{Q_{\rm D3}}{\rho} \ , \tag{5.11}$$

$h_{\rm D3}$ and $Q_{\rm D3}$ being yet new real integration constants. The last condition to determine where $H_{\rm D7}$ is generically only solvable term by term in its Laurent expansion. However, there are special cases that allow for a resummation of the resulting solution. For example, when the

[1] We adopt the string frame all throughout this chapter.

D5s are nearly cored (*i.e.* $h_{\rm D5} = 0$), (5.9) is solved by

$$H_{\rm D7} \;=\; \begin{cases} h_{\rm D7} + Q_{\rm D7}\left(x_1^2 - \dfrac{Q_5}{3}x_2^3\right) & , \\ \text{or} & \\ h_{\rm D7} + Q_{\rm D7}\left(x_1^2 + \dfrac{4Q_5}{9}x_2^3\right)^{-1/6} & , \end{cases} \tag{5.12}$$

in terms of two extra integration constants $h_{\rm D7}$ and $Q_{\rm D7}$. When $h_{\rm D5} \neq 0$, simple solutions are of course found by solving $\partial^2_{x_1} H_{\rm D7} = 0$ and $\partial^2_{x_2} H_{\rm D7} = 0$ separately. Otherwise, more non-trivial possibilities are

$$H_{\rm D7} \;=\; H^{(0)}_{\rm D7} + \frac{x_1^2}{2} - H^{(-2)}_{\rm D5} \,, \tag{5.13}$$

where the function $H^{(0)}_{\rm D7}$ satisfies $\partial^2_{x_1} H^{(0)}_{\rm D7} \;=\; \partial^2_{x_2} H^{(0)}_{\rm D7} \;=\; 0$, and $H^{(-2)}_{\rm D5}$ denotes the second primitive of $H_{\rm D5}$, *i.e.* satisfying $\partial^2_{x_2} H^{(-2)}_{\rm D5} \;=\; H_{\rm D5}$.

The class of solutions presented here does not contain any obvious AdS_3 solutions to be obtained by means of an appropriate near-horizon procedure. Nevertheless, they have a possible interesting physical interpretation as the gravity duals of the position dependent configurations of the coupling within $\mathcal{N} = 4$ SYM, which were studied in [23].

5.2 | D3 – D3′ – D7 brane systems

Let us now move to a different brane configuration in type IIB. Motivated by the SYM deformation with $\text{ISO}(1,1) \times \text{SO}(2) \times \text{SO}(4)$ symmetry and 2D spacetime dependent couplings obtained in [22], we propose the following $\frac{1}{8}$-BPS brane configuration.

Within the same D3 brane background, we place this time orthogonal D3 branes (denoted by D3′) as well as D7 branes in such a way as to leave $(1+1)$D Lorentz symmetry intact. This procedure, which is illustrated in detail in table 5.2, turns out to preserve four real supercharges by imposing

$$\Pi(\mathcal{O}_{\rm D3})\,\epsilon \;\overset{!}{=}\; \Pi(\mathcal{O}_{\rm D7})\,\epsilon \;\overset{!}{=}\; \Pi(\mathcal{O}_{\rm D3'})\,\epsilon \;\overset{!}{=}\; \epsilon \,, \tag{5.14}$$

we realize that the three projections are perfectly compatible and each of them halves the space of invariant eigenspinors in such a way that the resulting solution again depends on four real parameters. This means that this brane system is $\frac{1}{8}$ BPS, as in the previous section.

object	t	y	x_1	x_2	r	ϕ	ρ	θ_1	θ_2	θ_3
D3	×	×	×	×	–	–	–	–	–	–
D3′	×	×	∼	∼	×	×	–	–	–	–
D7	×	×	–	–	×	×	×	×	×	×

Table 5.2: The $\frac{1}{8}$ BPS brane system underlying the intersection of D3′ – D7 branes intersecting D3 branes.

Our Ansatz on the IIB fields is:

$$\begin{aligned} ds_{10}^2 &= H_{\mathrm{D3}}^{-1/2} H_{\mathrm{D3'}}^{-1/2} H_{\mathrm{D7}}^{-1/2} ds_{\mathrm{Mkw}_2}^2 + H_{\mathrm{D3}}^{-1/2} H_{\mathrm{D3'}}^{1/2} H_{\mathrm{D7}}^{1/2} \left(dx_1^2 + dx_2^2\right) \\ &\quad + H_{\mathrm{D3}}^{1/2} H_{\mathrm{D3'}}^{-1/2} H_{\mathrm{D7}}^{-1/2} \left(dr^2 + r^2 d\phi^2\right) + H_{\mathrm{D3}}^{1/2} H_{\mathrm{D3'}}^{1/2} H_{\mathrm{D7}}^{-1/2} \left(d\rho^2 + \rho^2 ds_{S^3}^2\right) \ , \qquad (5.15) \\ C_{(4)} &= \left(H_{\mathrm{D7}} H_{\mathrm{D3}}^{-1} \mathrm{vol}_{\mathrm{Mkw}_2} \wedge dx_1 \wedge dx_2 + H_{\mathrm{D3'}}^{-1} \mathrm{vol}_{\mathrm{Mkw}_2} \wedge \mathrm{vol}_{\mathbb{R}^2}\right)|_{\mathrm{SD}} \ , \qquad (5.16) \\ C_{(6)} &= 0 \ , \qquad (5.17) \\ C_{(8)} &= H_{\mathrm{D3}} H_{\mathrm{D7}}^{-1} \mathrm{vol}_{\mathrm{Mkw}_2} \wedge \mathrm{vol}_{\mathbb{R}^2} \wedge \mathrm{vol}_{\mathbb{R}^4} \ , \qquad (5.18) \\ e^{\Phi} &= H_{\mathrm{D7}}^{-1} \ , \qquad (5.19) \end{aligned}$$

where $ds_{\mathrm{Mkw}_2}^2 \equiv (-dt^2 + dy^2)$, $ds_{S^3}^2 \equiv (d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2)$, and $(\cdots)|_{\mathrm{SD}}$ denotes projection onto the self-dual field. The warp factors appearing in the above Ansatz are respectively assumed to have the following spacetime dependence: $H_{\mathrm{D3}} = H_{\mathrm{D3}}(r,\rho)$, $H_{\mathrm{D3'}} = H_{\mathrm{D3'}}(\rho)$ and $H_{\mathrm{D7}} = H_{\mathrm{D7}}(x_1, x_2)$.

The complete set of BI (2.54) and equations of motion (2.55) – (2.58) is satisfied upon imposing the following differential equations

$$\Delta_{\mathbb{R}^4} H_{\mathrm{D3}} + H_{\mathrm{D3'}} \Delta_{\mathbb{R}^2} H_{\mathrm{D3}} \equiv \rho^{-3}\partial_\rho\left(\rho^3 \partial_\rho H_{\mathrm{D3}}\right) + H_{\mathrm{D3'}} r^{-1} \partial_r \left(r \partial_r H_{\mathrm{D3}}\right) \overset{!}{=} 0 \ , \qquad (5.20)$$

$$\Delta_{\mathbb{R}^4} H_{\mathrm{D3'}} \equiv \rho^{-3}\partial_\rho\left(\rho^3 \partial_\rho H_{\mathrm{D3'}}\right) \overset{!}{=} 0 \ , \qquad (5.21)$$

$$\Delta_{\Sigma} H_{\mathrm{D7}} \equiv \partial_{x_1}^2 H_{\mathrm{D7}} + \partial_{x_2}^2 H_{\mathrm{D7}} \overset{!}{=} 0 \ . \qquad (5.22)$$

In this case the above system can be *e.g.* solved by[2]

$$\begin{aligned} H_{\mathrm{D3}} &= 1 + Q_{\mathrm{D3}} \left(\frac{1}{\rho^2} + \kappa_1 \log r + \kappa_2 \log\rho + \kappa_3 \left(\rho^2 - 2r^2\right)\right) \ , \qquad (5.23) \\ H_{\mathrm{D3'}} &= 1 + \frac{Q_{\mathrm{D3'}}}{\rho^2} \ , \qquad (5.24) \\ H_{\mathrm{D7}} &= 1 + Q_{\mathrm{D7}} \log\left(x_1^2 + x_2^2\right) \ , \qquad (5.25) \end{aligned}$$

where Q_{D3}, $Q_{\mathrm{D3'}}$, Q_{D7}, κ_i are real integration constants satisfying

$$\kappa_2 \overset{!}{=} 4 Q_{\mathrm{D3'}} \kappa_3 \ . \qquad (5.26)$$

Near-Horizon limit and AdS$_3$

As $\rho \to 0$, one has the following asymptotic behavior for the warp factors

$$H_{\mathrm{D3}} \sim \frac{Q_{\mathrm{D3}}}{\rho^2} \ , \quad \text{and} \quad H_{\mathrm{D3'}} \sim \frac{Q_{\mathrm{D3'}}}{\rho^2} \ , \qquad (5.27)$$

[2]Note that, depending on possible boundary conditions chosen in the (x_1, x_2) plane, H_{D7} can be an arbitrary harmonic function. The one we write here is the only rotationally invariant form for this function, but there are infinite different solutions.

while $H_{\rm D7}$ stays finite since it is a ρ-independent harmonic function of (x_1, x_2). In this limit, the solution in (5.15) – (5.19), after rescaling the Minkowski worldvolume[3], takes the form

$$ds_{10}^2 = \ell^2 H^{-1/2} \left(ds^2_{\rm AdS_3} + Q_{\rm D3}^{-1} H ds^2_\Sigma + Q_{\rm D3'}^{-1} ds^2_{\mathbb{T}^2} + ds^2_{S^3} \right) \ , \tag{5.28}$$

$$C_{(4)} = \ell^2 \rho^2 \, {\rm vol}_{\rm Mkw_2} \wedge \left({Q_{\rm D3}}^{-1} H {\rm vol}_\Sigma + Q_{\rm D3'}^{-1} {\rm vol}_{\mathbb{T}^2} \right) |_{\rm SD} \ , \tag{5.29}$$

$$C_{(6)} = 0 \ , \tag{5.30}$$

$$C_{(8)} = \ell^4 Q_{\rm D3} H^{-1} \, {\rm vol}_{\rm AdS_3} \wedge {\rm vol}_{\mathbb{T}^2} \wedge {\rm vol}_{S^3} \ , \tag{5.31}$$

$$e^\Phi = H^{-1} \ , \tag{5.32}$$

where $ds^2_{\rm AdS_3} \equiv \left(\frac{\rho^2}{\ell^2} ds^2_{\rm Mkw_2} + \frac{d\rho^2}{\rho^2} \right)$ is the metric of unit AdS$_3$, with ${\rm vol}_{\rm AdS_3} \equiv \ell^{-2} \rho \, dt \wedge dy \wedge d\rho$ the volume element, $ds^2_\Sigma \equiv (dx_1^2 + dx_2^2)$ and $ds^2_{\mathbb{T}^2} \equiv (dr^2 + r^2 d\phi^2)$, while the constant length ℓ is defined as $\ell \equiv (Q_{\rm D3} Q_{\rm D3'})^{1/4}$, and finally H is an arbitrary harmonic function on Σ.

The above solution represents AdS$_3 \times S^3 \times \mathbb{T}^2$ warped over a Riemann surface Σ. As usual in these cases, besides an enhancement of spacetime symmetry, such a near-horizon geometry also possesses *enhanced supersymmetry*, from 4 to 8 real supercharges.

Discussion

When turning off a D3 brane charge, namely either $H_{\rm D3}$ or $H_{\rm D3'}$ are constant functions, a global symmetry enhancement to an S^5 takes place and the resulting D3–D7 brane intersection exhibits the near horizon geometry AdS$_3 \times S^5 \times \mathbb{T}^2$. Precisely, a particular choice of the deformation parameters of the SO(2) $\times$ SO(4) field theory obtained in [22] allows for such symmetry enhancement, in such a way that the resulting action exhibits a rigid ISO(1, 1)$\times$SO(6) invariance [19, 20, 21].

Similarly, when turning off the D7 brane charge, the SO(2) $\times$ SO(4) remains and there is a SUSY enhancement to 16 supercharges [103]. These solutions have been studied in [104, 105].

Despite a D3 – D7 – D7′ construction with the same rigid symmetry has been explored, we have not found a supergravity solution without smearing along the (x_1, x_2) directions.

In light of the classification for probe branes in AdS$_5 \times S^5$ done in [103], it would be interesting to study a stack of N D3 branes within a D3′ – D7 background in the probe approximation.

5.3 | D3 – D5 – NS5 – D5′ – NS5′ – D7 brane systems

Let us now return to the first brane system that we considered. We argued that it did not admit any special near-horizon geometry featuring lower-dimensional AdS factors. However, it was already observed in [32] that more general intersections of this type do yield AdS$_3$ vacua in Type IIB supergravity with $\mathcal{N} = 2$ supersymmetry in 3D. The explicit brane setup considered there is given in table 5.3 and it is originally obtained by performing a single T-duality on

[3]We take $(t, y) \to \frac{1}{\ell}(t, y)$.

the (massive) Type IIA setup given by D2 – D6 – NS5 – D4 – D8 – KK5. In [32] both the supergravity brane solution and the dual 2d quivers are discussed.

object	t	y	x_1	x_2	r	ϕ_1	ϕ_2	ρ	θ_1	θ_2
D3	×	×	×	×	–	–	–	–	–	–
D5	×	×	∼	×	–	–	–	×	×	×
NS5	×	×	×	∼	–	–	–	×	×	×
D5′	×	×	×	∼	×	×	×	–	–	–
NS5′	×	×	∼	×	×	×	×	–	–	–
D7	×	×	–	–	×	×	×	×	×	×

Table 5.3: The $\frac{1}{8}$ BPS brane system underlying the intersection of D5 – NS5 – D5′ – NS5′ – D7 branes intersecting D3 branes. The ∼ denotes smearing directions.

Starting from the configuration showed in table 5.1, one might add D5 and NS5 branes as shown in table 5.3. Interestingly, even though one might expect this configuration to be way less supersymmetric than the previous one, in the end none of the new projections are really independent of the ones already taken above. As a result, a generic Killing spinor found earlier will also be a good spinor for the new intersection of branes, which is therefore *still* $\frac{1}{8}$ BPS.

We consider the same brane setup as in [32], but, as opposed to [32], we do not want to keep any link with the (massive) IIA description. As a result, we will obtain a generalization of the brane solution described there, where no isometric circle is left in order to perform the T-duality back to Type IIA. Our solution will be *genuinely* Type IIB. Our Ansatz on the IIB fields reads

$$
\begin{aligned}
ds_{10}^2 =\; & H_{\rm D3}^{-1/2}H_{\rm D5}^{-1/2}H_{\rm D5'}^{-1/2}H_{\rm D7}^{-1/2}ds^2_{\rm Mkw_2} + H_{\rm D3}^{-1/2}H_{\rm D7}^{1/2}\left(\frac{H_{\rm D5'}^{1/2}}{H_{\rm D5}^{1/2}}H_{\rm NS5}dx_1^2 + \frac{H_{\rm D5}^{1/2}}{H_{\rm D5'}^{1/2}}H_{\rm NS5'}dx_2^2\right) \\
& + H_{\rm D3}^{1/2}H_{\rm D7}^{-1/2}\left(\frac{H_{\rm D5}^{1/2}}{H_{\rm D5'}^{1/2}}H_{\rm NS5}\left(dr^2+r^2ds^2_{S^2}\right) + \frac{H_{\rm D5'}^{1/2}}{H_{\rm D5}^{1/2}}H_{\rm NS5'}\left(d\rho^2+\rho^2ds^2_{\tilde{S}^2}\right)\right)\,, \qquad (5.33)\\
C_{(4)} =\; & H_{\rm D7}H_{\rm D3}^{-1}\,{\rm vol}_{\rm Mkw_2}\wedge dx_1\wedge dx_2|_{\rm SD}\,, \qquad (5.34)\\
C_{(6)} =\; & \frac{H_{\rm D5'}H_{\rm NS5'}}{H_{\rm D5}}\,{\rm vol}_{\rm Mkw_2}\wedge dx_1\wedge{\rm vol}_{\tilde{\mathbb{R}}^3} - \frac{H_{\rm D5}H_{\rm NS5}}{H_{\rm D5'}}\,{\rm vol}_{\rm Mkw_2}\wedge dx_2\wedge{\rm vol}_{\mathbb{R}^3}\,, \qquad (5.35)\\
C_{(8)} =\; & H_{\rm D3}H_{\rm NS5}H_{\rm NS5'}H_{\rm D7}^{-1}\,{\rm vol}_{\rm Mkw_2}\wedge{\rm vol}_{\mathbb{R}^3}\wedge{\rm vol}_{\tilde{\mathbb{R}}^3}\,, \qquad (5.36)\\
B_{(6)} =\; & H_{\rm D7}\left(\frac{H_{\rm D5}H_{\rm NS5}}{H_{\rm NS5'}}\,{\rm vol}_{\rm Mkw_2}\wedge dx_1\wedge{\rm vol}_{\mathbb{R}^3} + \frac{H_{\rm D5'}H_{\rm NS5'}}{H_{\rm NS5}}\,{\rm vol}_{\rm Mkw_2}\wedge dx_2\wedge{\rm vol}_{\tilde{\mathbb{R}}^3}\right)\,, \qquad (5.37)\\
e^{\Phi} =\; & H_{\rm D5}^{-1/2}H_{\rm D5'}^{-1/2}H_{\rm NS5}^{1/2}H_{\rm NS5'}^{1/2}H_{\rm D7}^{-1}\,, \qquad (5.38)
\end{aligned}
$$

where $ds^2_{\rm Mkw_2} \equiv (-dt^2+dy^2)$, $ds^2_{S^2} \equiv (d\phi_1^2+\sin^2\phi_1 d\phi_2^2)$, $ds^2_{\tilde{S}^2} \equiv (d\theta_1^2+\sin^2\theta_1 d\theta_2^2)$, and $(\cdots)|_{\rm SD}$ denotes projection onto the self-dual field. The warp factors appearing in the above Ansatz are respectively assumed to have the following spacetime dependence: $H_{\rm D3}=H_{\rm D3}(r,\rho)$, $H_{\rm D5}=H_{\rm D5}(r)$, $H_{\rm NS5}=H_{\rm NS5}(r)$, $H_{\rm D5'}=H_{\rm D5'}(\rho)$, $H_{\rm NS5'}=H_{\rm NS5'}(\rho)$ and $H_{\rm D7}=H_{\rm D7}(x_1,x_2)$.

In order for the complete set of BI (2.54) and equations of motion (2.55) – (2.58) to be satisfied, the following identification turns out to be necessary[4]

$$H_{\rm NS5} \;=\; H_{\rm D5}\;, \qquad \text{and} \qquad H_{\rm NS5'} \;=\; H_{\rm D5'}\;. \tag{5.39}$$

After taking (5.39) into account, one is left with the following set of differential equations

$$\Delta_{\tilde{\mathbb{R}}^3} H_{\rm D3} + \frac{H^2_{\rm D5'}}{H^2_{\rm D5}} \Delta_{\mathbb{R}^3} H_{\rm D3} \;\equiv\; \rho^{-2}\partial_\rho\left(\rho^2\partial_\rho H_{\rm D3}\right) + \frac{H^2_{\rm D5'}}{H^2_{\rm D5}} r^{-2}\partial_r\left(r^2\partial_r H_{\rm D3}\right) \overset{!}{=} 0\;, \tag{5.40}$$

$$\Delta_{\mathbb{R}^3} H_{\rm D5} \;\equiv\; r^{-2}\partial_r\left(r^2\partial_r H_{\rm D5}\right) \overset{!}{=} 0\;, \tag{5.41}$$

$$\Delta_{\tilde{\mathbb{R}}^3} H_{\rm D5'} \;\equiv\; \rho^{-2}\partial_\rho\left(\rho^2\partial_\rho H_{\rm D5'}\right) \overset{!}{=} 0\;, \tag{5.42}$$

$$\Delta_\Sigma H_{\rm D7} \;\equiv\; \partial^2_{x_1} H_{\rm D7} + \partial^2_{x_2} H_{\rm D7} \overset{!}{=} 0\;. \tag{5.43}$$

Equations (5.40), (5.41) and (5.42) are respectively solved by

$$H_{\rm D3} = 1 + Q_{\rm D3}\left(\frac{1}{r} + \frac{\kappa}{\rho}\right)\;, \tag{5.44}$$

$$H_{\rm D5} = H_{\rm NS5} = \;1 + \frac{Q_5}{r}\;, \tag{5.45}$$

$$H_{\rm D5'} = H_{\rm NS5'} = \;1 + \frac{Q_5'}{\rho}\;, \tag{5.46}$$

while we leave $H_{\rm D7}$ as an unspecified harmonic function on Σ.

Near-Horizon limit and AdS$_3$

As $r \to 0$ (or identically $\rho \to 0$, since the whole solution is completely symmetric w.r.t. swapping $r \leftrightarrow \rho$), one has the following asymptotic behavior for the warp factors

$$H_{\rm D3} \;\sim\; \frac{Q_{\rm D3}}{r}\;, \quad \text{and} \qquad H_{\rm D5} \;=\; H_{\rm NS5} \;\sim\; \frac{Q_5}{r}\;, \tag{5.47}$$

while $H_{\rm D5'}$, $H_{\rm NS5'}$ and $H_{\rm D7}$ stay finite since they are r-independent harmonic functions on $\tilde{\mathbb{R}}^3$ and Σ, respectively. In this limit, the solution in (5.33) – (5.38), after rescaling the Minkowski worldvolume[5], takes the form

$$ds^2_{10} = \ell^2 H^{-1/2} K^{-1/2}\left(4ds^2_{\rm AdS_3} + \frac{HK}{Q_{\rm D3}Q_5} ds^2_\Sigma + \frac{K^2}{Q_5^2} ds^2_{\mathbb{T}^3} + ds^2_{S^2}\right)\;, \tag{5.48}$$

$$C_{(4)} = 4\ell^2 {Q_{\rm D3}}^{-1} H r\, \text{vol}_{\rm Mkw_2} \wedge \text{vol}_\Sigma|_{\rm SD}\;, \tag{5.49}$$

$$C_{(6)} = 4\ell^2 {Q_5}^{-1} K^2 r\, \text{vol}_{\rm Mkw_2} \wedge dx_1 \wedge \text{vol}_{\mathbb{T}^3} + 8\ell^4 Q_5 K^{-1} \text{vol}_{\rm AdS_3} \wedge dx_2 \wedge \text{vol}_{S^2}\;, \tag{5.50}$$

$$C_{(8)} = 8\ell^4 Q_{\rm D3} H^{-1} K\, \text{vol}_{\rm AdS_3} \wedge \text{vol}_{S^2} \wedge \text{vol}_{\mathbb{T}^3}\;, \tag{5.51}$$

$$B_{(6)} = 4\ell^2 H\left({Q_5}^{-1} K^2 r\, \text{vol}_{\rm Mkw_2} \wedge dx_2 \wedge \text{vol}_{\mathbb{T}^3} - 2\ell^2 Q_5 K^{-1} \text{vol}_{\rm AdS_3} \wedge dx_1 \wedge \text{vol}_{S^2}\right)\;, \tag{5.52}$$

[4]It is worth noticing that these conditions represent the most general way of solving for the dynamics of the system if one aims at retaining a completely general (x_1, x_2) dependence of $H_{\rm D7}$.

[5]We take $(t, y) \to \frac{1}{2\ell}(t, y)$.

$$e^{\Phi} = H^{-1} , \tag{5.53}$$

where $ds^2_{\text{AdS}_3} \equiv \frac{Q_5}{\ell}\frac{r}{\ell}ds^2_{\text{Mkw}_2} + \frac{dr^2}{4r^2}$ is the metric of unit AdS$_3$, with $\text{vol}_{\text{AdS}_3} \equiv \frac{Q_5}{2\ell^2}\, dt \wedge dy \wedge dr$ the volume element, $ds^2_{\Sigma} \equiv (dx_1^2 + dx_2^2)$ and $ds^2_{\mathbb{T}^3} \equiv \left(d\rho^2 + \rho^2 ds^2_{\tilde{S}^2}\right)$, while the constant length ℓ is defined as $\ell \equiv (Q_{\text{D3}}Q_5^3)^{1/4}$, and finally H & K are two arbitrary harmonic functions on Σ and $\mathbb{T}^3$, respectively.

The above solution preserves 8 real supercharges and represents AdS$_3 \times S^2$ warped over $\mathbb{T}^3 \times \Sigma$. In particular, for a spherically symmetric choice for the function K, the resulting geometry can be thought of as AdS$_3 \times S^2 \times \tilde{S}^2$ warped over $I_\rho \times \Sigma$.

Discussion

The solution (5.48) interpreted as AdS$_3 \times S^2 \times \tilde{S}^2$ warped over $I_\rho \times \Sigma$ is a straightforward generalization of the one discussed in [32].[6] The generalization consists in taking H as a general harmonic function on Σ, rather than just being linear in one of the two coordinates, say x_1. As a consequence, since we have no Abelian isometries left in our background, we conclude that our setup is *genuinely* type IIB, with no relation to (massive) type IIA any longer. On the other hand though, the physical interpretation proposed there is directly carried through in our more general situation. Let us summarize in what follows the salient steps.

In particular, one may use the existence of a consistent truncation of type IIB supergravity on $S^2 \times \Sigma$ [106] to land within minimal $\mathcal{N} = (1, 1)$ $D = 6$ gauged supergravity [107]. This theory possesses 16 real supercharges and SU(2) gauge symmetry, and is often referred to as *Romans' supergravity*. Its field content is given by the 6d metric, a real scalar field X, a 2-form gauge potential $\mathcal{B}_{(2)}$, three SU(2) vector fields plus one Abelian vector field. The scalar potential of the theory admits a real superpotential formulation given as

$$V(X) \;=\; 16\,\left(-5f(X)^2 + X^2\,(D_X f)^2\right) \,, \tag{5.54}$$

where $f(X) \equiv \frac{1}{8}\,(3X + X^{-3})$. Note that the theory admits a SUSY AdS$_6$ extremum at $X = 1$ in the absence of vectors and 2-form.

Now, by using the uplift formulae in [106], one can show that, upon specifying a choice of harmonic function on Σ, the SUSY vacuum of the minimal 6D theory lifts to the class of AdS$_6$ vacua described in [108, 109, 110], where a brane interpretation is provided in terms of (p, q)5 brane webs and D7 branes. Furthermore, 6D BPS slicings of the form AdS$_3 \times S^2$ were studied in [15]. The authors of [32] were able to map the solutions obtained when lifting these configurations to type IIB to the near-horizon limit of D3 – D5 – NS5 – D5′ – NS5′ – D7 brane systems for which H_{D7} is purely linear in x_1.

This implies that our solution (5.48) may be interpreted holographically as the gravity dual of a defect CFT$_2$ inside a 5d $\mathcal{N} = 1$ SCFT, which is engineered by placing within the background (p, q)5 web and D7 branes, D3 branes and a new (p, q)5 web only sharing two directions with the previous one. The more general (x_1, x_2) dependence present in our solution w.r.t. the one in [32] simply suggests that the AdS$_6$ vacuum, which is reached asymptotically when moving

[6] Let us note that, in contrast with the brane intersection of [32], in this solution the D7 brane can be lobectomized taking $H_{\text{D7}} \to 1$.

at infinite distance from the defect branes, will no longer necessarily correspond to the *annulus* as a choice for Σ.

Chapter 6

Open Strings in type IIB orientifold reductions in $d = 6$

In this chapter, we explore Type IIB compactifications on general 4D group manifolds, considering both O-planes and D-branes that fill the six-dimensional spacetime. By turning on background fluxes consistent with the orientifold projection, the lower-dimensional description emerges as a 6D $\mathcal{N} = (1,1)$ gauged supergravity. We show how the dynamical open strings on the D-branes are incorporated by introducing additional vector multiplets and embedding tensor deformations, which modify the usual bulk field strengths and ensure consistency with the source-corrected Bianchi identities. These modifications can be viewed as U-dual versions of the Green-Schwarz terms. Finally, we show that the resulting scalar potential in six dimensions exactly matches the dimensional reduction of the ten-dimensional bulk action, once the contributions from O-planes and D-branes are properly taken into account.

6.1 | Bulk Reduction

In order to carry out the dimensional reduction of type IIB down to six dimensions, we parametrize the ten-dimensional metric $g_{\mathcal{MN}}$ in terms of the six-dimensional one and the moduli describing the four-dimensional internal metric. In particular, by picking

$$ds^2_{(10)} = g_{\mathcal{MN}}\, dx^{\mathcal{M}} \otimes dx^{\mathcal{N}} = \tau^{-2}\, g^{(6)}_{\mu\nu}\, dx^{\mu} \otimes dx^{\nu} + \rho\, ds^2_{(4)} \ , \tag{6.1}$$

the universal moduli ρ and τ are singled out, whereas the rest of the moduli are encoded inside $g^{(4)}$ and describe volume preserving deformations of the internal geometry. In addition to that, we introduce local indices m, n as

$$ds^2_{(4)} = \mathcal{M}_{mn}\, v^m \otimes v^n \ , \tag{6.2}$$

where the matrix $\mathcal{M}_{mn}$ parametrizes the coset $\mathrm{SL}(4,\mathbb{R})/\mathrm{SO}(4)$ and $\det \mathcal{M} = 1$.

To obtain the 6D gravity action in the Einstein frame upon compactification, we require the constraint [111]

$$\rho^2 \stackrel{!}{=} e^{2\Phi}\, \tau^4 \ , \tag{6.3}$$

which implies that ρ and τ fix the internal volume and the string coupling.

Let us consider now each of the terms in the type IIB effective action. The determinant of the metric reduces to

$$\sqrt{-g} \quad \rightarrow \quad \tau^{-6}\,\rho^{2}\,\sqrt{g_{(4)}}\,\sqrt{g_{(6)}}\ . \tag{6.4}$$

Then, the reduction of the Einstein term in (2.52) amounts to [1]

$$\begin{aligned}\int d^{10}x\,\sqrt{-g}\,e^{-2\Phi}\mathcal{R}^{(10)} \quad &\rightarrow \quad \int d^{10}x\,\sqrt{-g_{(6)}}\,\left(\tau^{-4}\,\rho^{2}\,e^{-2\Phi}\,\mathcal{R}^{(6)} + \tau^{-6}\,\rho\,e^{-2\Phi}\,\mathcal{R}^{(4)}\right)\\ &= \quad \int d^{10}x\,\sqrt{-g_{(6)}}\,\left(\mathcal{R}^{(6)} - V_{\omega}\right)\ ,\end{aligned} \tag{6.5}$$

where $V_{\omega} \equiv -\rho^{-1}\,\tau^{-2}\,\mathcal{R}^{(4)}$. In case of twisted toroidal compactifications, where v^{m} are the Maurer-Cartan 1-forms, $\mathcal{R}^{(4)}$ has the following expression [112]:

$$\mathcal{R}^{(4)} = -\frac{1}{4}\,\mathcal{M}_{mq}\mathcal{M}^{nr}\mathcal{M}^{ps}\,\omega_{np}{}^{q}\,\omega_{rs}{}^{m} - \frac{1}{2}\,\mathcal{M}^{np}\,\omega_{mn}{}^{q}\,\omega_{qp}{}^{m}\ , \tag{6.6}$$

where $\mathcal{M}^{mn}$ is the inverse of $\mathcal{M}_{mn}$ and $\omega_{mn}{}^{p}$ are the structure constants entering the Maurer-Cartan equation

$$dv^{m} + \frac{1}{2}\omega_{np}{}^{m}\,v^{n}\wedge v^{p} = 0\ . \tag{6.7}$$

This, in turn, implies the Jacobi identities as an integrability condition,

$$\omega_{[mn}{}^{r}\,\omega_{p]r}{}^{q} = 0\ . \tag{6.8}$$

In addition to this, we will ask the structure constants to fulfill the unimodularity condition $\omega_{mn}{}^{n} = 0$ for consistency, as we are performing the compactification at the level of the action.

Let us consider now the scalar potential arising from the H flux. Reducing the corresponding term of the action (2.52) yields

$$\int d^{10}x\,\sqrt{-g}\left(-\frac{1}{12}\,e^{-2\Phi}\,|H_{(3)}|^{2}\right) \quad \rightarrow \quad \int d^{6}x\,\sqrt{-g_{(6)}}\left(-\frac{1}{12}\,H_{mnp}\,H^{mnp}\,\rho^{-3}\,\tau^{-2}\right)\ , \tag{6.9}$$

so that the contribution consists of $V_{H} \equiv \frac{1}{12}\,H_{mnp}\,H^{mnp}\,\rho^{-3}\,\tau^{-2}$ and the contraction with the indices is done with the internal metric $g^{(4)}$.

Regarding the R-R p-forms[2], their contribution to the scalar potential is

$$\int d^{10}x\,\sqrt{-g}\left(-\frac{1}{2p!}\,|F_{(p)}|^{2}\right) \quad \rightarrow \quad \int d^{6}x\,\sqrt{-g_{(6)}}\left(-\frac{1}{2p!}\,F_{m_1\cdots m_p}\,F^{m_1\cdots m_p}\,\rho^{2-p}\,\tau^{-6}\right)\ , \tag{6.10}$$

so that $V_{F_p} = \frac{1}{2p!}\,F_{m_1\cdots m_p}\,F^{m_1\cdots m_p}\,\rho^{2-p}\,\tau^{-6}$.

Thus, as the 10D Chern-Simons term does not give any contribution to the potential, the

[1] Please note that $\mathcal{R}^{(10)} \rightarrow \tau^{2}\,\mathcal{R}^{(6)} + \rho^{-1}\,\mathcal{R}^{(4)}$.

[2] Please note that the numerical factor in (2.52) is $-\frac{1}{4p!}$ due to the simultaneous presence of the dual magnetic fields $F_{(10-p)}$. Using the duality relations (2.53), the factor $\frac{1}{2p!}$ is trivially restored.

reduced 6D theory is given by the following action

$$S_{6\mathrm{D}} = \int d^6x\sqrt{-g^{(6)}}\,\left(\mathcal{R}^{(6)} + 2\mathcal{L}_{\mathrm{kin}} - V\right)\ , \tag{6.11}$$

where the full scalar potential arising from the bulk and the effective tension consists of

$$V = V_\omega + V_H + \sum_p V_{F_p}\ . \tag{6.12}$$

The kinetic term for the moduli, which span a $\mathbb{R}^+_\rho \times \mathbb{R}^+_\tau \times \mathrm{SL}(4,\mathbb{R})/\mathrm{SO}(4)$ geometry, is given by

$$\mathcal{L}_{\mathrm{kin}} = -\frac{(\partial\rho)^2}{4\rho^2} - \frac{(\partial\tau)^2}{\tau^2} + \frac{1}{8}\mathrm{Tr}\left(\partial\mathcal{M}\partial\mathcal{M}^{-1}\right)\ . \tag{6.13}$$

6.2 | Non-Abelian Brane Actions and Reductions Thereof

6.2.1. Case O5/D5 & Open Strings

Let us compute the contribution of the sources to the scalar potential. Focusing on the scalar sector and making use of eqs. (6.91) and (6.92), we find that the reduction of the DBI action of N_{D5} coincident D5-branes is given by

$$\begin{aligned} S^{\mathrm{DBI}}_{\mathrm{D5}} =& -N_{\mathrm{D5}}\,T_{\mathrm{D5}}\int d^6x\sqrt{-g}\left[\rho^{-1}\tau^{-4}\left(1+\frac{\lambda^2}{6}g_{IJK}\epsilon_{mnpq}h^qY^{Im}Y^{Jn}Y^{Kp}\right)\right.\\ &\left.+\frac{\lambda^2}{4}\rho\tau^{-4}g_{IJ}{}^{M}g_{KLM}M_{mn}M_{pq}Y^{Im}Y^{Jp}Y^{Kn}Y^{Lq}+\dots\right]\ , \end{aligned} \tag{6.14}$$

where the dots mean subleading contributions in λ (or, equivalently, in α') which cannot be captured by the gauged-supergravity description together with other terms that will not enter the scalar potential. On the other hand, for the O5 planes we get simply the contribution from the tension

$$S^{\mathrm{DBI}}_{\mathrm{O5}} = -N_{\mathrm{D5}}\,T_{\mathrm{O5}}\int d^6x\sqrt{-g}\rho^{-1}\tau^{-4} + \dots\ . \tag{6.15}$$

Hence, the total contribution from the DBI action of the sources amounts to

$$\begin{aligned} V^{\mathrm{DBI}}_{\mathrm{D5/O5}} =& \rho^{-1}\tau^{-4}\left[2\kappa_6^2\left(N_{\mathrm{D5}}\,T_{\mathrm{D5}}+T_{\mathrm{O5}}\right)+\frac{2\kappa_6^2\lambda^2N_{\mathrm{D5}}\,T_{\mathrm{D5}}}{6}g_{IJK}\epsilon_{mnpq}h^qY^{Im}Y^{Jn}Y^{Kp}\right]\\ &+\rho\tau^{-4}\left(\frac{2\kappa_6^2\lambda^2N_{\mathrm{D5}}\,T_{\mathrm{D5}}}{4}g_{IJ}{}^{M}g_{KLM}M_{mn}M_{pq}Y^{Im}Y^{Jp}Y^{Kn}Y^{Lq}\right)\ . \end{aligned} \tag{6.16}$$

This is not the only contribution of the sources to the scalar potential, as the Wess-Zumino terms in the D5-brane action give additional contributions. These, however, have been already included through the modification of the field strength (6.95). Let us discuss this aspect in more detail. The Wess-Zumino terms in the D5-brane action include a coupling to $C_{(8)}$,

$$S_{\rm D5}^{\rm WZ} = \mu_{\rm D5} \int_{\rm WV(D5)} {\rm Tr}\left({\rm P}\left[e^{i\lambda \iota_Y \iota_Y} \hat{C}_{(8)} \wedge e^{\hat{B}_{(2)}} \wedge e^{\lambda \mathcal{F}}\right]\right) + \ldots\,, \tag{6.17}$$

which modifies the Bianchi identity of $C_{(0)}$ and consequently the form of the associated field strength. Locally, we now have $F_{(1)} = dC_{(0)} + \chi_{(1)}$ for some 1-form $\chi_{(1)}$. The effect of this in our setup is that now the $F_{(1)}$-flux is no longer constant,

$$F_m = f_m + \Delta f_m\,, \tag{6.18}$$

since Δf_m is a certain combination of the non-Abelian scalars Y^{Im}. This will result in two additional contributions to the scalar potential coming from the kinetic term of $F_{(1)}$, namely:

$$V_{F_{(1)}} = \frac{\rho\,\tau^{-6}}{2} M^{mn} \left(f_m + \Delta f_m\right)\left(f_n + \Delta f_n\right)\,. \tag{6.19}$$

By a standard argument, the term in the potential which is linear in Δf_m can be read by evaluating (6.17) using the uncorrected expression for $\hat{C}_{(8)}$.[3] Let us do this explicitly in order to show how one can get from a direct calculation the expression of the modified field strength presented in the main text, (6.95).

First, we expand the integrand of (6.17) at the relevant order in λ:

$$\begin{aligned} \mu_{\rm D5} \int {\rm Tr}\left({\rm P}\left[e^{i\lambda \iota_Y \iota_Y} \hat{C}_{(8)} \wedge e^{\hat{B}_{(2)}} \wedge e^{\lambda \mathcal{F}}\right]\right) &= i\lambda \mu_{\rm D5} \int {\rm Tr}\, \iota_Y \iota_Y \hat{C}_{(8)} + \ldots \\ = -\frac{i\lambda\mu_{\rm D5}}{2!6!} \int dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_6}\, {\rm Tr}\left(\hat{C}_{(8)\mu_1\ldots\mu_6 mn}[Y^m, Y^n]\right) + \ldots \end{aligned} \tag{6.20}$$

Now we insert the expression of $\hat{C}_{(8)}$, which is the following[4]

$$\hat{C}_{(8)\mu_1\ldots\mu_6 mn} = \frac{\lambda}{3}\, \epsilon_{\mu_1\ldots\mu_6 mnp}{}^{q} f_q Y^p\,. \tag{6.21}$$

Plugging this in (6.20) yields[5]

$$\begin{aligned} &\mu_{\rm D5} \int {\rm Tr}\left({\rm P}\left[e^{i\lambda \iota_Y \iota_Y} \hat{C}_{(8)} \wedge e^{\hat{B}_{(2)}} \wedge e^{\lambda \mathcal{F}}\right]\right) = \\ &= \frac{\lambda^2 N_{\rm D5}\, \mu_{\rm D5}}{3!} \int d^6x \sqrt{-g}\rho\tau^{-6}\, g_{IJK} \epsilon_{mnpq} M^{qr} f_r Y^{Im} Y^{Jn} Y^{Kp} + \ldots\,. \end{aligned} \tag{6.22}$$

Comparing this with the term in the scalar potential (6.19) which is linear in Δf_m, we obtain

$$\Delta f_m = -\frac{2\kappa_6^2 \lambda^2 N_{\rm D5}\, \mu_{\rm D5}}{3!} g_{IJK} \epsilon_{mnpq} Y^{In} Y^{Jp} Y^{Kq}\,, \tag{6.23}$$

[3] Namely, the one obtained solving $dC_{(8)} = \star dC_{(0)}$, with $C_{(0)}$ given by (6.80).

[4] Our conventions are such that $\epsilon_{01\ldots d-1} = +\sqrt{-g^{(d)}}$.

[5] $\epsilon_{1234} = +\sqrt{\det M_{mn}} = +1$.

as anticipated in (6.95).

6.2.2. Case O7/D7 & Open Strings

In this section we will explain how to obtain the modified field strengths $F_{(1)}$ and $F_{(3)}$ in (6.130) and (6.131) from the WZ effective actions. We will study the case of $F_{(1)}$ and give similar arguments for $F_{(3)}$.

Let us firstly consider the contribution of the $C_{(8)}$ potential in the bulk and the WZ actions.[6]

Using the type IIB democratic formulation (2.52), together with the duality relation $F_{(9)} = \star F_{(1)}$, the variation of

$$S = S_{\text{IIB}} + S^{\text{WZ}}_{\text{D7}} + S^{\text{WZ}}_{\text{O7}} \tag{6.24}$$

with respect to $C_{(8)}$ is

$$\delta S_{\text{total}} = \int_{10} \left(\frac{1}{2\kappa_{10}^2}\, d \star F_{(9)} + (\star J)_{(2)} \right) \wedge \delta C_{(8)} \ , \tag{6.25}$$

where we have rewritten the WZ action as

$$S^{\text{WZ}}_{\text{O}p/\text{D}p} = \int_{\text{WV}(\text{O}p/\text{D}p)} \omega^{(p+1)} = \int_{10} \omega^{(p+1)} \wedge \delta^{\text{O}p/\text{D}p}_{9-p} \ , \tag{6.26}$$

with $\delta^{\text{D}p}_{9-p} \equiv \delta(x^{p+1}) \cdots \delta(x^{p+1})\, dx^{p+1} \wedge \cdots \wedge dx^9$. The quantity $(\star J)_{(2)}$ is defined through the WZ action as follows:

$$S^{\text{WZ}}_{\text{D7}} = \int_{10} C_{(8)} \wedge (\star J)_{(2)} \ . \tag{6.27}$$

Therefore, the $C_{(8)}$ equation of motion becomes the modified Bianchi identity for $F_{(1)}$,

$$dF_{(1)} = -\, 2\, \kappa_8^2\, (\star J)_{(2)} \ , \tag{6.28}$$

in such a way the source term is straightforwardly determined from the $C_{(8)}$ couplings in the expansion of the WZ action. On the other hand, taking into account that the product $\sigma_{\text{O7}}(-1)^{F_L}\Omega$ on the components $(C_{\mu_0\cdots\mu_5 ab}, C_{\mu_0\cdots\mu_5 ai}, C_{\mu_0\cdots\mu_5 ij})$ is, respectively, $(+,-,+)$, only the quantities $(\star J)_{ab}$ and $(\star J)_{ij}$ will be nonzero. Here, because the flux $\overline{F}_a$ is allowed while $\overline{F}_i$ is projected out, we will focus on $(\star J)_{ab}$.

In this case, we have to consider the following $C_{(8)}$ couplings:

$$\begin{aligned} S^{\text{WZ}}_{\text{D7}} =& \mu_{\text{D7}} \int \text{Tr} \Big(\text{P}[\hat{C}_{(8)}] + i\lambda^2 \text{P}[\iota_Y \iota_Y \hat{C}_{(8)}] \wedge \mathcal{F}\frac{1}{2} \\ & \qquad -\frac{1}{2}\lambda^2 \text{P}[(\iota_Y \iota_Y)^2 \hat{C}_{(8)} \wedge \hat{B} \wedge \hat{B}] + \ldots \Big) \ , \end{aligned} \tag{6.29}$$

$$S^{\text{WZ}}_{\text{O7}} = \mu_{\text{O7}} \int C_{(8)} + \ldots \ . \tag{6.30}$$

[6] Note that the DBI action does not contribute to the equation of motion of the RR fields.

The first contribution can be conveniently rewritten as

$$\int \mathrm{Tr}\left(\mathrm{P}[\hat{C}_{(8)}]\right) = \int \frac{1}{6!2!}\,\mathrm{Tr}\left(\mathrm{P}[\hat{C}_{(8)}]_{\mu_0\cdots\mu_5\underline{ab}}\right)\,\delta(y^1)\,\delta(y^2)\,dx^{\mu_0\cdots\mu_5\underline{ab}}\wedge\frac{\epsilon_{\underline{ij}}}{2}\,dy^{\underline{ij}}\,, \tag{6.31}$$

where we have used the notation $dx^{M_1\cdots M_n}\equiv dx^{M_1}\wedge\cdots\wedge dx^{M_n}$.

The pullback of the 8-form potential is expressed as

$$\mathrm{P}[\hat{C}_{(8)}]_{\mu_0\cdots\mu_5\underline{ab}} = \hat{C}_{\mu_0\cdots\mu_5\underline{ab}} - \lambda D_{[\mu_0}Y^{\underline{k}}\,\hat{C}_{\mu_1\cdots\mu_5\underline{ab}]\underline{k}} + \frac{\lambda^2}{2}D_{[\mu_0}Y^{\underline{k}}\,D_{\mu_1}Y^{\underline{l}}\,\hat{C}_{\mu_2\cdots\mu_5\underline{ab}]\underline{kl}} + \dots\ , \tag{6.32}$$

where, moreover, $\hat{C}_{(8)}$ is Taylor-expanded around the position of the source, $y^i=0$,

$$\hat{C}_{\mu_0\cdots\mu_5 ab} = C_{\mu_0\cdots\mu_5 ab} + \lambda\,Y^{\underline{i}}\,\partial_{\underline{i}}\,C_{\mu_0\cdots\mu_5\underline{ab}} + \frac{\lambda^2}{2}\,Y^{\underline{i}}\,Y^{\underline{j}}\,\partial_{\underline{i}}\,\partial_{\underline{j}}\,C_{\mu_0\cdots\mu_5\underline{ab}} + \dots\ . \tag{6.33}$$

Here, $C_{\mu_0\cdots\mu_5\underline{ab}}$ is a 10D field, which admits the Kaluza Klein decomposition

$$C_{\mu_0\cdots\mu_5\underline{ab}}(x,y) = C_{\mu_0\cdots\mu_5 mn}(x)\,v^m{}_{\underline{a}}(y)\,v^n{}_{\underline{b}}(y) + \dots\ , \tag{6.34}$$

where ellipses account for other nontrivial terms entering the compactification Ansatz that do not depend on the 8-form potential which will be omitted in this analysis without loss of generality.

All in all, we find that the first term in (6.32) gives the following contribution to $(\star J)_{ab}$:

$$\hat{C}_{\mu_0\cdots\mu_5\underline{ab}} = v^a{}_{\underline{a}}\,v^b{}_{\underline{b}}\,\frac{\lambda^2}{2}\,Y^{Ii'}\,Y^{Jj'}\,\omega_{ai'}{}^k\,\omega_{bj'}{}^l\,C_{\mu_0\cdots\mu_5 kl}\,t_I\,t_J\ . \tag{6.35}$$

If we multiply both sides of the equation by $\epsilon_{\underline{ij}}$ and use the Schouten identity on the $\underline{kl}$ and $\underline{ij}$ indices, we have

$$\hat{C}_{\mu_0\cdots\mu_5\underline{ab}}\epsilon_{ij} = v^a{}_{\underline{a}}\,v^b{}_{\underline{b}}\,\frac{\lambda^2}{2}\,Y^{Ii'}\,Y^{Jj'}\,\omega_{ai'}{}^k\,\omega_{bj'}{}^l\,C_{\mu_0\cdots\mu_5 ij}\epsilon_{kl}\,t_I\,t_J\ . \tag{6.36}$$

Let us consider now the second term of (6.32). Using the twist matrices (6.121), this term contains the quantity

$$\begin{aligned} -\,\lambda D_{[\mu_0}Y^{\underline{k}}\,\hat{C}_{\mu_1\cdots\mu_5\underline{ab}]\underline{k}} = &-v^a{}_{\underline{a}}\,v^b{}_{\underline{b}}\,\frac{8\cdot 7}{2}\,\frac{\lambda^2}{2}\,g_{JK}{}^I \\ &\times\mathcal{A}^J{}_{[a|}\,Y^{Kk}Y^{Li}\left(\omega_{i|b]}{}^j\,C_{\mu_0\cdots\mu_5 jk} + \omega_{ik}{}^c\,C_{\mu_0\cdots\mu_5|b]c}\right)t_I\,t_L + \dots\ . \end{aligned} \tag{6.37}$$

Then, while the first term contributes to $(\star J)_{ab}$ (we need to use the Schouten identity on the indices $[\underline{ijk}]$ and take the trace), the latter does not, due to the presence of the longitudinal indices $[bc]$ in the potential. In particular, the full term is given by

$$\mathrm{Tr}\left(-\lambda D_{[\mu_0}Y^{\underline{k}}\,\hat{C}_{\mu_1\cdots\mu_5\underline{ab}]\underline{k}}\right) = v^a{}_{\underline{a}}\,v^b{}_{\underline{b}}\,\frac{8\cdot 7}{4}\,N_{\mathrm{D7}}\,\frac{\lambda^2}{2}\,g_{JKL}\,\eta_{[a}\,\mathcal{A}^J{}_{b]}\,Y^{Kk}Y^{Li}\,C_{\mu_0\cdots\mu_5 ki} + \dots\ . \tag{6.38}$$

The last term in (6.32) can be written as

$$\frac{\lambda^2}{2} D_{[\mu_0} Y^{\underline{k}} D_{\mu_1} Y^{\underline{l}} \hat{C}_{\mu_2\cdots\mu_5\underline{ab}]\underline{kl}} = v^a{}_{\underline{a}}\, v^b{}_{\underline{b}}\, \frac{8\cdot 7}{2}\,\frac{\lambda^2}{2} \mathcal{A}^J{}_{[a|}\, \mathcal{A}^{J'}{}_{|b]}\, Y^{Kk}\, Y^{K'l} \times g_{JK}{}^I\, g_{J'K'}{}^{I'}\, C_{\mu_0\cdots\mu_5 kl}\, t_I\, t_{I'} + \ldots \; . \tag{6.39}$$

Multiplying by $\epsilon_{\underline{ij}}$ and using again the Schouten identity, we have

$$\frac{\lambda^2}{2} D_{[\mu_0} Y^{\underline{k}} D_{\mu_1} Y^{\underline{l}} \hat{C}_{\mu_2\cdots\mu_5\underline{ab}]\underline{kl}}\, \epsilon_{ij} = v^a{}_{\underline{a}}\, v^b{}_{\underline{b}}\, \frac{8\cdot 7}{2}\,\frac{\lambda^2}{2} \mathcal{A}^J{}_{[a|}\, \mathcal{A}^{J'}{}_{|b]}\, Y^{Kk}\, Y^{K'l}\, g_{JK}{}^I\, g_{J'K'}{}^{I'}\, C_{\mu_0\cdots\mu_5 ij}\, \epsilon_{kl}\, t_I\, t_{I'} + \ldots \; , \tag{6.40}$$

where we have omitted terms proportional to D_μ.

Next, by studying the second term coming from the WZ action (6.29), we observe that the only contribution to $(\star J)_{ab}$ arises from

$$\mathrm{Tr}\left(i\lambda^2\, \mathrm{P}[\iota_Y\, \iota_Y\, \hat{C}_{(8)}] \wedge \mathcal{F}\right) = -N_{\mathrm{D}7}\, \frac{\lambda^2}{2}\, \kappa_{II'}\, g_{JK}{}^I\, g_{J'K'}{}^{I'} \times \mathcal{A}^I{}_a\, \mathcal{A}^J{}_b\, Y^{I'i}\, Y^{J'j}\, \epsilon_{ij}\, C_{\mu_0\cdots\mu_5 kl} \frac{dx^{\mu_0\cdots\mu_5}}{6!} \wedge \frac{v^{kl}}{2!} \wedge \frac{v^{ab}}{2!} + \ldots \; , \tag{6.41}$$

where we have used the notation $v^{mn} \equiv v^m \wedge v^n$. Precisely, this term cancels the one in (6.40).

Finally, let us consider the third and last term in the WZ action (6.29), $\mathrm{P}[(\iota_Y\, \iota_Y)^2\, \hat{C}_{(8)}]$. This consists of the 4th interior product over the vector $Y^{\underline{i}}$ of a 12-form. Because we are dealing with codimension-2 objects, this turns out to be trivially zero.

Therefore, according to (6.27), the final expression for $(\star J)_{ab}$ is

$$(\star J)_{ab} = N_{\mathrm{D}7}\, \mu_{\mathrm{D}7}\, \frac{\lambda^2}{2} \left(\kappa_{II'}\, \epsilon_{kl} Y^{Ii'}\, Y^{I'j'}\, \omega_{ai'}{}^k\, \omega_{bj'}{}^l + g_{JKL}\, \eta_{[a}\, \mathcal{A}^J{}_{b]}\, Y^{Kk} Y^{Li} \epsilon_{ki}\right) \; , \tag{6.42}$$

which implies, using (6.28),

$$dF_{(1)} = -2\,\kappa_8^2\, N_{\mathrm{D}7}\, \mu_{\mathrm{D}7}\, \frac{\lambda^2}{2\cdot 2!} \Big(\kappa_{II'}\, \epsilon_{kl} Y^{Ii'}\, Y^{I'j'}\, \omega_{ai'}{}^k\, \omega_{bj'}{}^l + g_{JKL}\, \eta_{[a}\, \mathcal{A}^J{}_{b]}\, Y^{Kk} Y^{Li} \epsilon_{ki}\Big)\; v^a \wedge v^b + \ldots \; . \tag{6.43}$$

Then, because generically its internal part is $F_{(1)} = F_a\, v^a + F_i\, v^i$ and $\omega_{ab}{}^i = 0$, the only contribution to $(\star J)_{ab}$ arises from F_a. In particular,

$$dF_{(1)} = -\frac{1}{2} F_a \left(\omega_{ij}{}^a\, v^i \wedge v^j + \omega_{bc}{}^a\, v^b \wedge v^c\right) + \ldots \; . \tag{6.44}$$

Equating the $v^a \wedge v^b$ components with (6.43) and using the Jacobi identities (6.109), in particular the first equation in (6.133), we obtain

$$F_a = \frac{\tilde{\lambda}_7^2}{2} \left(\kappa_{ai}{}^j\, \kappa_{IJ}\, Y^{Ii}\, Y^{Jk} \epsilon_{jk} - g_{JKL}\, \mathcal{A}^J{}_a\, Y^{Kk} Y^{Li} \epsilon_{ki}\right) + \Delta_a \; , \qquad \epsilon^{ab}\, \Delta_a\, \eta_b = 0 \; , \tag{6.45}$$

where $\tilde{\lambda}_7 \equiv (2\kappa_8^2 T_{\mathrm{D7}})^{1/2}\lambda$. Precisely, the compactification Ansatz for $C_{(0)}$, together with the integrability condition (6.111) allow us to identify $\Delta_a = \overline{F}_a$, in such a way that (6.130) is recovered.

A similar argument applies to the field strength $F_{(3)}$ in (6.131). In this case we need to study the couplings to $C_{(6)}$ in the WZ action, so that we can read off the current $(\star J)_{abij}$, which is defined via this expression:

$$S_{\mathrm{D7}}^{\mathrm{WZ}} = \int_{10} C_{(6)} \wedge (\star J)_{(4)} \,. \tag{6.46}$$

Namely, as we are interested in the couplings to $C_{\mu_0\cdots\mu_5}$, from the WZ action we have the following contributions:

$$\begin{aligned} S_{\mathrm{D7}}^{\mathrm{WZ}} = \mu_{\mathrm{D7}} \int \mathrm{Tr}\,\Big(& \mathrm{P}[\hat{C}_{(6)} \wedge \hat{B}_{(2)}] + \lambda\,\mathrm{P}[\hat{C}_{(6)}] \wedge \mathcal{F} + \frac{i}{2}\lambda\,\mathrm{P}[\iota_Y\iota_Y(\hat{C}_{(6)} \wedge \hat{B}_{(2)}^2)] \\ & + \frac{i}{2}\lambda^2\,\mathrm{P}[\iota_Y\iota_Y(\hat{C}_{(6)} \wedge \hat{B}_{(2)})] \wedge \mathcal{F} - \frac{1}{2}\lambda^2\,\mathrm{P}[(\iota_Y\iota_Y)^2(\hat{C}_{(6)} \wedge \hat{B}_{(2)}^3)]\Big) + \dots \,. \end{aligned} \tag{6.47}$$

The two terms in the second line do not contribute to the current: While the latter is trivially zero because the D7 has codimension 2, the former is $\mathcal{O}(\lambda^2)$ and the scalars B_{ij} are projected out by the O7 plane. Then, according to (6.131) and the compactification Ansatz for $\hat{B}_{(2)}$, it is expected that only the first and second terms give nontrivial contributions.

6.2.3. Case O9/D9 & Open Strings

Let us firstly consider the D9/O9 contributions to the scalar potential. The two-derivative action of a stack of N_{D9} D9-branes is given by $S_{\mathrm{D9}} = S_{\mathrm{D9}}^{\mathrm{DBI}} + S_{\mathrm{D9}}^{\mathrm{WZ}}$, where

$$S_{\mathrm{D9}}^{\mathrm{DBI}} = -\,N_{\mathrm{D9}}\,T_{\mathrm{D9}} \int d^{10}x\,\sqrt{-g}\,e^{-\Phi}\left(1 + \frac{\lambda^2}{4}\,\mathcal{F}^I{}_{\mu\nu}\mathcal{F}^{I\,\mu\nu}\right) + \dots \,, \tag{6.48}$$

$$S_{\mathrm{D9}}^{\mathrm{WZ}} = \mu_{\mathrm{D9}} \int \mathrm{Tr}\left(C_{(10)} + \frac{\lambda^2}{2}\,C_{(6)} \wedge \mathcal{F} \wedge \mathcal{F}\right) + \dots \,. \tag{6.49}$$

On the other hand, the O9-plane contribution is

$$S_{\mathrm{O9}} = -\,T_{\mathrm{O9}} \int d^{10}x\,\sqrt{-g}\,e^{-\Phi} + \mu_{\mathrm{O9}} \int C_{(10)} + \dots \,, \tag{6.50}$$

where the O9-plane charge is $\mu_{\mathrm{O9}} = 32\,\epsilon_{\mathrm{O9}}\,\mu_{\mathrm{D9}}$. The tadpole cancellation condition,

$$N_{\mathrm{D9}}\mu_{\mathrm{D9}} + \mu_{\mathrm{O9}} = 0\,, \tag{6.51}$$

requires $N_{\mathrm{D9}} = 32$ and $\epsilon_{\mathrm{O9}} = -1$, which corresponds to an O9$^-$ plane. Hence, the total contribution from the sources, $S_{\mathrm{D9/O9}} = S_{\mathrm{D9}} + S_{\mathrm{O9}}$, amounts to

$$S_{\mathrm{D9/O9}} = -\lambda^2 N_{\mathrm{D9}} T_{\mathrm{D9}} \int d^{10}x \sqrt{-g} \left[\frac{e^{-\Phi}}{4} \mathcal{F}^I_{\mu\nu} \mathcal{F}^{I\mu\nu} + \frac{1}{2 \cdot 6! \cdot (2!)^2} \epsilon^{\mu_1 \dots \mu_{10}} C_{(6)\mu_1 \dots \mu_6} \mathcal{F}^I{}_{\mu_7\mu_8} \mathcal{F}_{I\mu_9\mu_{10}} \right], \tag{6.52}$$

where adjoint indices are lowered using the Cartan-Killing metric, $\mathcal{F}_I = \kappa_{IJ}\mathcal{F}^J$. Taking into account the supergravity fields that survive the O9 projection (see Table 6.6), we can write down the full action as

$$\begin{aligned} S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \Big\{ & e^{-2\Phi} \left[\mathcal{R} + 4(\partial\Phi)^2 \right] - \frac{1}{2 \cdot 7!} \left| F_{(7)} \right|^2 - \frac{\tilde{\lambda}_9^2}{4} e^{-\Phi} \mathcal{F}^I_{\mu\nu} \mathcal{F}^{I\mu\nu} \\ & - \frac{\tilde{\lambda}_9^2}{2 \cdot 6! \cdot (2!)^2} \epsilon^{\mu_1 \dots \mu_{10}} C_{(6)\mu_1 \dots \mu_6} \mathcal{F}^I{}_{\mu_7\mu_8} \mathcal{F}_{I\mu_9\mu_{10}} \Big\}, \end{aligned} \tag{6.53}$$

where

$$\tilde{\lambda}_9^2 \equiv 2\kappa_{10}^2 \lambda^2 N_{\mathrm{D9}}\, T_{\mathrm{D9}}\,. \tag{6.54}$$

This is nothing but the bosonic effective action of type I string theory, written in terms of a RR 6-form potential $C_{(6)}$. In order to write it down in the standard form, we dualize it into a 2-form potential $C_{(2)}$. To this aim, we integrate by parts the last term in (6.53) and introduce a Lagrange multiplier, as usual. Using differential-form notation, we have

$$\begin{aligned} S \to S' &= S - \frac{\tilde{\lambda}_9^2}{4\kappa_{10}^2} \int d\left(C_{(6)} \wedge \Omega_{(3)}\right) + \frac{1}{2\kappa_{10}^2} \int F_{(7)} \wedge dC_{(2)} \\ &= \frac{1}{2\kappa_{10}^2} \int \left\{ \frac{1}{2} F_{(7)} \wedge \star F_{(7)} + F_{(7)} \wedge \left(dC_{(2)} - \frac{\tilde{\lambda}_9^2}{2} \Omega_{(3)} \right) \right\} + \dots \\ &= \frac{1}{2\kappa_{10}^2} \int \left\{ \frac{1}{2} F_{(7)} \wedge \star F_{(7)} + F_{(7)} \wedge F_{(3)} \right\}, \end{aligned} \tag{6.55}$$

where we have defined the Chern-Simons 3-form $\Omega_{(3)}$,

$$\Omega_{(3)} = \mathcal{F}^I \wedge \mathcal{A}^I + \frac{1}{3!} g_{IJK}\, \mathcal{A}^I \wedge \mathcal{A}^J \wedge \mathcal{A}^K\,, \qquad d\Omega_{(3)} = \mathcal{F}^I \wedge \mathcal{F}_I\,, \tag{6.56}$$

as well as the modified field strength $F_{(3)}$,

$$\begin{aligned} F_{(3)} &= dC_{(2)} - \frac{\tilde{\lambda}_9^2}{2} \Omega_{(3)} \\ &= dC_{(2)} - \frac{\tilde{\lambda}_9^2}{2} \left(\mathcal{F}^I \wedge \mathcal{A}^I + \frac{1}{3!} g_{IJK}\, \mathcal{A}^I \wedge \mathcal{A}^J \wedge \mathcal{A}^K \right). \end{aligned} \tag{6.57}$$

The variation of S' with respect to $F_{(7)}$ (now considered non-dynamical) gives

$$F_{(3)} = - \star F_{(7)}\,. \tag{6.58}$$

whereas the variation with respect to $C_{(2)}$ gives the Bianchi identity of $F_{(7)}$, namely $dF_{(7)} = 0$. The Bianchi identity of $F_{(3)}$ is now modified as a consequence of the coupling of the open string-sector to $C_{(6)}$. It reads

$$dF_{(3)} = -\frac{\tilde{\lambda}_9^2}{2}\mathcal{F}^I \wedge \mathcal{F}_I \,. \tag{6.59}$$

Finally, we substitute the duality relation (6.58) back into the action. This yields the bosonic action of type I supergravity, as anticipated:

$$S' = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left\{e^{-2\Phi}\left[\mathcal{R} + 4(\partial\Phi)^2\right] - \frac{1}{2\cdot 3!}\left|F_{(3)}\right|^2 - \frac{\tilde{\lambda}_9^2}{4}e^{-\Phi}\mathcal{F}^I_{\mu\nu}\mathcal{F}^{I\mu\nu}\right\}. \tag{6.60}$$

When considering the reduction Ansatz (6.138) and (6.140), the internal components of the field strength $\mathcal{F}^I$ are

$$\mathcal{F}^I = \frac{1}{2}\left(\overline{\mathcal{F}}^I{}_{mn} - g_{JK}{}^I\,\mathcal{A}^J{}_m\,\mathcal{A}^K{}_n - \mathcal{A}^I{}_p\,\omega_{mn}{}^p\right)v^m \wedge v^n + \ldots\ , \tag{6.61}$$

where we have assumed that

$$g_{IJK}\,\sigma^I = 0\ , \qquad\qquad d\sigma^I = \frac{1}{2}\,\overline{\mathcal{F}}^I{}_{mn}\,v^m \wedge v^n\ , \tag{6.62}$$

for constant $\overline{\mathcal{F}}^I{}_{mn}$. The latter imposes the integrability condition

$$\overline{\mathcal{F}}^I{}_{q[m}\,\omega_{np]}{}^q = 0\ , \tag{6.63}$$

which turns out to be a quadratic constraint in supergravity (6.152).

Similarly, the internal components of $F_{(3)}$ result

$$\begin{aligned} F_{mnp} = \overline{F}_{mnp} - 3\,C_{q[m}\,\omega_{np]}{}^q & \\ - \tilde{\lambda}_9^2\left(3\,\mathcal{A}^I{}_m\,\overline{\mathcal{F}}^I{}_{np} - g_{IJK}\,\mathcal{A}^I{}_m\,\mathcal{A}^J{}_n\,\mathcal{A}^K{}_p - \frac{3}{2}\,\omega_{mn}{}^q\,\mathcal{A}^I{}_p\,\mathcal{A}^I{}_q\right)&\ , \end{aligned} \tag{6.64}$$

where we have introduced

$$d\gamma - \frac{\tilde{\lambda}_9^2}{2}\sigma^I \wedge d\sigma^I \equiv \frac{1}{3!}\,\overline{F}_{mnp}v^m \wedge v^n \wedge v^p\ , \tag{6.65}$$

for constant $\overline{F}_{mnp}$. This leads to the integrability condition

$$\frac{\tilde{\lambda}_9^2}{2}\,\overline{\mathcal{F}}^I{}_{[mn}\,\overline{\mathcal{F}}^I{}_{pq]} - \overline{F}_{r[mn}\,\omega_{pq]}{}^r = 0\ , \tag{6.66}$$

which is again a quadratic constraint, (6.152).

6.3 | Gauged $\mathcal{N}=(1,1)$ Supergravities in 6D

Ungauged $\mathcal{N}=(1,1)$ supergravity stems from dimensional reduction of type I supergravity on a $\mathbb{T}^4$. In this case, the complete set of closed string zero mode excitations is contained in the coupling between the gravity multiplet and four vector multiplets. Since the goal of this chapter is that of using $\mathcal{N}=(1,1)$ supergravities as a tool for studying type IIB orientifold reductions including an excited open string sector, we need to introduce their general formulation featuring the coupling with an arbitrary number of vector multiplets[7]. The (ungauged) theory enjoys the following global symmetry

$$G_{\text{global}} \;=\; \mathbb{R}^+ \,\times\, \text{SO}(4,4+\mathfrak{N}) \;, \tag{6.67}$$

where $\mathfrak{N}$ is the number of extra vector multiplets. The physical (propagating) dof's of the theory are suitably rearranged into irrep's of the little group and of the global symmetry group as described in Table 6.1.

6D fields	$\text{SO}(4)=\text{SU}(2)_{\text{L}}\times\text{SU}(2)_{\text{R}}$	$\mathbb{R}^+_\Sigma\,\times\,\text{SO}(4,4+\mathfrak{N})$ irrep's	# dof's
$g_{\mu\nu}$	$(\mathbf{3},\mathbf{3})$	$\mathbf{1}^{(0)}$	9
$\mathcal{B}_{\mu\nu}$	$(\mathbf{1},\mathbf{3})\oplus(\mathbf{3},\mathbf{1})$	$\mathbf{1}^{(+2)}$	6
$\mathcal{A}_\mu^{\;M}$	$(\mathbf{2},\mathbf{2})$	$\square^{(-1)}\equiv(\mathbf{8}+\mathfrak{N})^{(-1)}$	$4(8+\mathfrak{N})$
Σ	$(\mathbf{1},\mathbf{1})$	$\mathbf{1}^{(-1)}$	1
$\mathcal{V}_M{}^{\underline{M}}$	$(\mathbf{1},\mathbf{1})$	$\mathbf{adj}^{(0)}$	$4(4+\mathfrak{N})$

Table 6.1: *The* $(64{+}8\mathfrak{N})_B$ *bosonic dof's of the theory arranged into irrep's of* $\text{SO}(4)_{little}\times G_{global}$*, the internal global symmetry being the one defined in* (6.67)*. Note that, within the scalars transforming in the adjoint, one should subtract the compact generators to get the correct number of propagating dof's.*

In particular, the $17{+}4\mathfrak{N}$ scalar fields of the theory parametrize the following coset geometry

$$\mathcal{M}_{\text{scalar}} \;=\; \underbrace{\mathbb{R}^+}_{\Sigma} \times \underbrace{\frac{\text{SO}(4,4+\mathfrak{N})}{\text{SO}(4)\times\text{SO}(4+\mathfrak{N})}}_{\mathcal{H}_{MN}} \;, \tag{6.68}$$

where the scalar coset representative $\mathcal{H}_{MN}$ is written in terms of a vielbein $\mathcal{V}_M{}^{\underline{M}}$ as

$$\mathcal{V}_M{}^{\underline{M}}\mathcal{V}_N{}^{\underline{M}} \;\equiv\; \mathcal{V}_M{}^{\underline{m}}\mathcal{V}_N{}^{\underline{m}} \,+\, \mathcal{V}_M{}^{\underline{\hat{m}}}\mathcal{V}_N{}^{\underline{\hat{m}}} \,+\, \mathcal{V}_M{}^{\underline{I}}\mathcal{V}_N{}^{\underline{I}} \;=\; \mathcal{H}_{MN} \;, \tag{6.69}$$

where the local $\text{SO}(4)\times\text{SO}(4+\mathfrak{N})$ index $\underline{M}$ has been split into $(\underline{m},\underline{\hat{m}},\underline{I})$, related to its $\text{SO}(4)_{\text{timelike}}$, $\text{SO}(4)_{\text{spacelike}}$, and $\text{SO}(\mathfrak{N})$ parts, respectively. The kinetic Lagrangian is given by

$$\mathcal{L}_{\text{kin}} \;=\; -2\Sigma^{-2}(\partial\Sigma)^2 \,+\, \frac{1}{16}\,\partial\mathcal{H}_{MN}\partial\mathcal{H}^{MN} \;. \tag{6.70}$$

The deformations of the ungauged theory which are consistent with bosonic symmetry as

[7]We denote this number by $(4+\mathfrak{N})$, where the first 4 are needed in order to describe the closed string sector, while the extra $\mathfrak{N}$ accounts for the number of vector multiplets associated to the open string sector, just as appearing in equation (3.17).

well as supersymmetry can arranged into the following embedding tensor irrep's

$$\Theta \;\in\; \underbrace{\square^{(+3)}}_{\zeta_M} \oplus \underbrace{\square^{(-1)}}_{\xi_M} \oplus \underbrace{\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}^{(-1)}}_{f_{[MNP]}} \;, \tag{6.71}$$

where ζ_M corresponds to a massive deformation inducing a Stückelberg coupling for the two-form $\mathcal{B}_{\mu\nu}$, while the remaining two irreducible pieces are traditional gaugings. In particular, f_{MNP} purely gauges a subgroup of $\mathrm{SO}(4, 4+\mathfrak{N})$, whereas ξ_M gauges a combination of the $\mathbb{R}^+_\Sigma$ generator and generators in the $\mathrm{SO}(4, 4+\mathfrak{N})$ part.

Now, given a specification of the embedding tensor Θ transforming as in (6.71), the consistency of the deformed theory demands its gauge invariance, which is enforced by imposing the following set of quadratic constraints (QC)

$$\begin{array}{rclcrcl} 3\, f_{R[MN}\, f_{PQ]}{}^{R} \;-\; 2\, f_{[MNP}\, \xi_{Q]} & = & 0 \;, & \qquad & \zeta_{(M}\, \xi_{N)} & = & 0 \;, \\ f_{MNP}\, \zeta^{P} \;-\; \xi_{[M}\, \zeta_{N]} & = & 0 \;, & & \xi_M\, \xi^M & = & 0 \;, \\ f_{MNP}\, \xi^{P} & = & 0 \;, & & \zeta_M\, \xi^M & = & 0 \;, \end{array} \tag{6.72}$$

which include conditions for the closure of the gauge algebra, *i.e.* generalized Jacobi identities. In (6.72), contractions are defined by means of the invariant $\mathrm{SO}(4, 4+\mathfrak{N})$ metric η_{MN} and its inverse η^{MN}. In what follows, we will perform a lightcone (LC) basis choice within the $\mathrm{SO}(4,4)$ sector, combined with a standard Cartesian basis along the remaining $\mathrm{SO}(\mathfrak{N})$ directions. The explicit form of η in this case is

$$\eta_{MN} \;=\; \left(\begin{array}{c|c|c} \mathbb{O}_4 & \mathbb{I}_4 & \mathbb{O}_{4,\mathfrak{N}} \\ \hline \mathbb{I}_4 & \mathbb{O}_4 & \mathbb{O}_{4,\mathfrak{N}} \\ \hline \mathbb{O}_{\mathfrak{N},4} & \mathbb{O}_{\mathfrak{N},4} & \mathbb{I}_{\mathfrak{N}} \end{array}\right) \;. \tag{6.73}$$

It is perhaps worth mentioning that this choice of basis precisely matches the one made in [52] within the $(4,4)$ part, which will represent the closed string sector of our type IIB orientifold compactifications. This choice is justified by the fact that closed string background fluxes have a natural mapping into LC components of the embedding tensor.

Embedding tensor deformations turn out to induce Yukawa-like couplings between scalars and fermions, which are parametrized by the so-called fermionic shift matrices. As a consequence, supersymmetry invariance of the action requires the presence of a scalar potential, which turns out to be quadratic in Θ. Its explicit form in terms of embedding tensor irrep's reads

$$\begin{aligned} V \;=\; \frac{g^2}{4}\bigg[& f_{MNP}\, f_{QRS}\Sigma^{-2}\left(\frac{1}{12}\mathcal{H}^{MQ}\mathcal{H}^{NR}\mathcal{H}^{PS} - \frac{1}{4}\mathcal{H}^{MQ}\eta^{NR}\eta^{PS} + \frac{1}{6}\eta^{MQ}\eta^{NR}\eta^{PS}\right) \\ & + \frac{1}{2}\zeta_M\zeta_N\Sigma^6\mathcal{H}^{MN} \;+\; \frac{2}{3}f_{MNP}\zeta_Q\Sigma^2\mathcal{H}^{MNPQ} \;+\; \frac{5}{4}\xi_M\xi_N\Sigma^{-2}\mathcal{H}^{MN}\bigg] \;, \end{aligned} \tag{6.74}$$

where $\mathcal{H}^{MN}$ denotes the inverse of $\mathcal{H}_{MN}$ and g is the gauge coupling constant. For simplicity, in the remainder of the chapter, we fix $g = 2$. The four-index antisymmetric object $\mathcal{H}^{MNPQ}$

appearing above is instead defined through

$$\mathcal{H}_{MNPQ} \equiv \epsilon_{\underline{mnpq}} \mathring{\mathcal{V}}_M{}^{\underline{m}} \mathring{\mathcal{V}}_N{}^{\underline{n}} \mathring{\mathcal{V}}_P{}^{\underline{p}} \mathring{\mathcal{V}}_Q{}^{\underline{q}} \ , \tag{6.75}$$

in terms of the Cartesian vielbein $\mathring{\mathcal{V}}_M{}^{\underline{M}}$, which is in turn related to the LC one $\mathcal{V}_M{}^{\underline{M}}$ through $\mathring{\mathcal{V}}_M{}^{\underline{M}} = \mathcal{V}_M{}^{\underline{N}} U_{\underline{N}}{}^{\underline{M}}$, with

$$U_{\underline{M}}{}^{\underline{N}} = \left(\begin{array}{c|c|c} -\frac{1}{\sqrt{2}}\mathbb{I}_4 & \frac{1}{\sqrt{2}}\mathbb{I}_4 & \mathbb{O}_{4,\mathfrak{N}} \\ \hline \frac{1}{\sqrt{2}}\mathbb{I}_4 & \frac{1}{\sqrt{2}}\mathbb{I}_4 & \mathbb{O}_{4,\mathfrak{N}} \\ \hline \mathbb{O}_{\mathfrak{N},4} & \mathbb{O}_{\mathfrak{N},4} & \mathbb{I}_{\mathfrak{N}} \end{array} \right) \ , \tag{6.76}$$

transforming the LC metric into $\text{diag}(-\mathbb{I}_4, +\mathbb{I}_4, \mathbb{I}_{\mathfrak{N}})$.

In [52] all possible orientifold reductions yielding $\mathcal{N} = (1, 1)$ theories in six dimensions were studied within the closed string sector. The closed string dynamics turned out to be contained within the theories with only the four (universal) vector multiplets included. In the next sections we will select the type IIB cases of interest and include an excited open string sector. This will require analyzing the 6D supergravity theories in the form presented in this section, *i.e.* with the inclusion of $\mathfrak{N}$ extra vector multiplets. Such an extension will allow us to study open string dof's like brane position moduli and/or Wilson line moduli, *i.e.* axions arising from internal legs of the worldvolume gauge fields. Moreover, we will be able to consider possible physical effects of a non-Abelian worldvolume theory, and/or the presence of worldvolume flux wrapping internal space.

6.4 | O5/D5 & Open Strings

Let us start by considering the minimal possible spacetime filling O-planes that respect 6D Lorentz symmetry, *i.e.* O5-planes. These are placed as follows within 10D spacetime,

$$\text{O5} \ : \ \underbrace{\times \ \times \ \times \ \times \ \times \ \times}_{\text{6D spacetime}} \ | \ \underbrace{- \ - \ - \ -}_{y^m} \ , \qquad \sigma_{\text{O5}} \ : \ y^m \ \longrightarrow \ -y^m \ ,$$

where σ_{O5} is the orientifold involution, whose action flips the sign of all transverse coordinates. The O5 projection is realized at the level of the 10D supergravity fields by means of the simultaneous action of the aforementioned involution, together with the worldsheet parity operator. Such a procedure yields the correct field content of a half-maximal supergravity in 6D. The resulting details of this projection are collected in Table 6.2.

In this case, due to presence of O5's and D5's, the reduction Ansatz can be formulated in a $\text{SL}(4,\mathbb{R}) \times G_{\text{YM}}$ covariant way. The 10D bulk supergravity Ansatz containing the 17 closed string scalars reads

$$ds^2_{(10)} = \tau^{-2}\, g_{\mu\nu} dx^\mu dx^\nu \ + \ \rho\, M_{mn}\, dy^m dy^n \ , \tag{6.77}$$

$$e^\Phi = \rho\, \tau^{-2} \ , \tag{6.78}$$

$$B_{(2)} = \frac{1}{6}\, \epsilon_{mnpq}\, h^m y^n dy^p \wedge dy^q \ + \ \dots \ , \tag{6.79}$$

IIB fields	σ_{O5}	Ω	# dof's
$e^m{}_n$	$+$	$+$	$16 - 6 = 10$
B_{mn}	$+$	$-$	—
Φ	$+$	$+$	1
$C_{(0)}$	$+$	$-$	—
C_{mn}	$+$	$+$	6
C_{mnpq}	$+$	$-$	—
Y^{Im}	$-$	$-$	$4\mathfrak{N}$

Table 6.2: *The $\mathbb{Z}_2$ parity of all internal components of the different IIB fields in the presence of spacetime filling O5-planes. The allowed ones yield excitable 6D scalar fields. Note that the total amount of resulting scalars correctly gives* $17 + 4\mathfrak{N}$, i.e. *the dimension of the supergravity coset* (6.68).

$$C_{(0)} = f_m\, y^m \, + \, \dots \, , \tag{6.80}$$

$$C_{(2)} = \frac{1}{(2!)^2}\, \epsilon_{mnpq}\, \gamma^{mn} dy^p \wedge dy^q \, + \, \dots \, , \tag{6.81}$$

$$C_{(4)} = 0 \, + \, \dots \, , \tag{6.82}$$

where "$+\dots$" denotes that we are discarding the terms in the Ansatz that do not contribute to the scalar potential. The scalars ρ, τ represent the volume and dilaton would-be moduli, M_{mn} is an element of $\text{SL}(4,\mathbb{R})/\text{SO}(4)$ describing deformations of the internal metric, and γ^{mn} is antisymmetric and contains the scalars coming from the R-R two-form $C_{(2)}$. These modes add up to 17, as they should. The remaining $4\mathfrak{N}$ scalars are part of the open-string sector and are denoted as Y^{Im}, where the index I labels the adjoint representation of G_{YM}.

On the other hand, h^m and f_m are constants parametrizing the $\overline{H}_{(3)}$ and $\overline{F}_{(1)}$ fluxes within the closed string sector,

$$\overline{F}_m \;=\; f_m \;, \qquad \overline{H}_{mnp} \;=\; \epsilon_{mnpq}\, h^q \;, \tag{6.83}$$

while in the open-string one we have the possibility of considering a non-Abelian gauge group with structure constants $g_{IJ}{}^K$. The consistency requirements on the aforementioned flux parameters purely reduce to the Jacobi identity for the Yang-Mills structure constants,

$$g_{[IJ}{}^{I'}\, g_{K]I'}{}^{L} \;\overset{!}{=}\; 0 \;. \tag{6.84}$$

Note that the flux tadpole induced by $H_{(3)}$ and $F_{(1)}$ does not have to vanish, since the Bianchi identity for $C_{(2)}$ gets modified by the presence of O5/D5 sources:

$$\underbrace{dF_{(3)}}_{=0} - \underbrace{H_{(3)} \wedge F_{(1)}}_{\neq 0} \;=\; j_{(4)}^{\text{O5/D5}} \;, \tag{6.85}$$

where $j_{(4)}^{\text{O5/D5}} = Q_5\, \text{vol}_{\mathcal{M}_4}$ is the effective current density. In the case at hands, we have that

$$Q_5 = 2\kappa^2 \left(N_{\text{D5}}\, \mu_{\text{D5}} + \mu_{\text{O5}}\right) = 2\kappa^2 \mu_{\text{D5}} \left(N_{\text{D5}} + 2\epsilon_{\text{O5}}\right) \, . \tag{6.86}$$

In the last equality we have made use of the relation between the D5 and O5 charges given in (3.11). The sign $\epsilon_{\mathrm{O5}} = \pm 1$ precisely determines the type of $\mathrm{O5}^{\pm}$ plane that we are considering.[8] The tadpole condition imposes the following condition

$$h^m f_m \overset{!}{=} Q_5 = 2\kappa^2 T_{\mathrm{D5}} \left(N_{\mathrm{D5}} + 2\epsilon_{\mathrm{O5}}\right) , \tag{6.87}$$

which must be taken into account when matching the scalar potential of the compactification with that of supergravity, as the latter only knows about the fluxes.

The scalar potential of the compactification ignoring the open-string sector was previously computed in [52]. Now we build on their results and also take into account open-string effects. The worldvolume action of the D5 branes contains two pieces: the DBI and the WZ actions. The first directly gives a contribution to the scalar potential, while the contribution of the second secretly appears through the bulk scalar potential given in section 6.1. The reason lies in the fact that the WZ action contains couplings between the open-string fields and the R-R potentials which in turn give rise to modified (bulk) field strengths associated to the dual R-R potentials.

Let us consider each contribution separately, first focusing on the one coming from the DBI. The DBI action of the D5 branes is given by (see section 6.2):

$$S_{\mathrm{D5}}^{\mathrm{DBI}} = -T_{\mathrm{D5}} \int d^6x \, \mathrm{Tr}\left(e^{-\hat{\Phi}} \sqrt{-\det(\mathbb{M}_{MN}) \det(\mathbb{Q}^i{}_j)}\right) , \tag{6.88}$$

where $T_{\mathrm{D5}} = 2\pi \ell_s^{-6}$ is the D5 brane tension. The indices $M, N, \ldots$ are worldvolume indices whereas $i, j, \ldots$ denote the transverse ones. The matrices $\mathbb{M}$ and $\mathbb{Q}$ are defined as

$$\mathbb{M}_{MN} = \mathrm{P}\left[\hat{E}_{MN} + \hat{E}_{Mi}(\mathbb{Q}^{-1} - \delta)^{ij} \hat{E}_{jN}\right] + \lambda \mathcal{F}_{MN} , \tag{6.89}$$

$$\mathbb{Q}^i{}_j = \delta^i{}_j + i\lambda [Y^i, Y^k] \hat{E}_{kj} , \tag{6.90}$$

where $E_{\mathcal{MN}} = \hat{g}_{\mathcal{MN}} + \hat{B}_{\mathcal{MN}}$.[9] Making use of the above compactification Ansatz, one finds that the matrices $\mathbb{M}$ and $\mathbb{Q}$ are given by

$$\mathbb{M}_{\mu\nu} = \tau^{-2} g_{\mu\nu} + \ldots , \tag{6.91}$$

$$\mathbb{Q}^m{}_n = \delta^m_n - \lambda \rho \, g_{IJ}{}^K Y^{Im} Y^{Jp} M_{pn} t_K + \frac{\lambda^2}{3} \epsilon_{npqr} h^r \, g_{IJ}{}^K Y^{Im} Y^{Jp} Y^{Lq} t_K t_L + \ldots , \tag{6.92}$$

where now the dots mean that we are ignoring terms that do not contribute to the scalar potential and also the ones which are of higher-order in λ. The generators of G_{YM} are denoted by t_I, and our conventions are such that $[t_I, t_J] = i \, g_{IJ}{}^K \, t_K$. Making use of (6.91) and (6.92)

[8] We refer to eq. (**??**) for further details.

[9] $\mathcal{M}, \mathcal{N}, \ldots$ are ten-dimensional indices and the meaning of the hat on ten-dimensional fields is explained in (**??**).

in (6.88), we obtain the following contribution to the scalar potential,

$$\begin{aligned} V_{\rm D5}^{\rm DBI} =& \rho^{-1}\tau^{-4}\left[2\kappa_6^2 N_{\rm D5}T_{\rm D5} + \frac{2\kappa_6^2\lambda^2 N_{\rm D5}T_{\rm D5}}{6} g_{IJK}\epsilon_{mnpq}h^q Y^{Im}Y^{Jn}Y^{Kp}\right] \\ &+ \rho\tau^{-4}\left(\frac{2\kappa_6^2\lambda^2 N_{\rm D5}T_{\rm D5}}{4} g_{IJ}{}^M g_{KLM} M_{mn}M_{pq}Y^{Im}Y^{Jp}Y^{Kn}Y^{Lq}\right), \end{aligned} \tag{6.93}$$

where $2\kappa_6^2 = 16\pi G_6$, being G_6 the six-dimensional Newton's constant.

In addition to (6.93), we will have the analogous contribution from the O5. Since the latter is non-dynamical, its contribution merely reduces to tension term (namely, the first one in (6.93)). Hence, the total DBI contribution from both types of sources is

$$\begin{aligned} V_{\rm O5/D5}^{\rm DBI} =& \rho^{-1}\tau^{-4}\left[2\kappa_6^2\left(N_{\rm D5}T_{\rm D5} + T_{\rm O5}\right) + \frac{2\kappa_6^2\lambda^2 N_{\rm D5}T_{\rm D5}}{6} g_{IJK}\epsilon_{mnpq}h^q Y^{Im}Y^{Jn}Y^{Kp}\right] \\ &+ \rho\tau^{-4}\left(\frac{2\kappa_6^2\lambda^2 N_{\rm D5}T_{\rm D5}}{4} g_{IJ}{}^M g_{KLM} M_{mn}M_{pq}Y^{Im}Y^{Jp}Y^{Kn}Y^{Lq}\right), \end{aligned} \tag{6.94}$$

On the other hand, the Wess-Zumino action contains a coupling between the scalars Y^{Ii} and $C_{(8)}$, see (6.17). As shown in section 6.2.1, this can be understood through a modified field strength F_m of the form,

$$F_m = \overline{F}_m - \frac{\tilde{\lambda}_5^2}{3!}\,\epsilon_{mnpq}\,g_{IJK}\,Y^{In}\,Y^{Jp}\,Y^{Kq}\,, \tag{6.95}$$

where $\tilde{\lambda}_5 = (2\kappa_6^2 N_{\rm D5}T_{\rm D5})^{1/2}\lambda$ and $g_{IJK} = g_{IJ}{}^L\,\kappa_{LK}$. Crucially now F_m is not pure flux, as it has the second contribution from the open-string scalars. On the contrary, the field strength H_{mnp} is not modified, so it simply reads

$$H_{mnp} = \overline{H}_{mnp}\,. \tag{6.96}$$

Finally, all that is left is to take the expression for the bulk scalar potential computed in [52] (see also section 6.1) and replace, according to (6.95),

$$\overline{F}_m \to \overline{F}_m - \frac{\tilde{\lambda}_5^2}{3!}\,\epsilon_{mnpq}\,g_{IJK}\,Y^{In}\,Y^{Jp}\,Y^{Kq}\,. \tag{6.97}$$

The resulting expression has to be added to (6.94), which yields the following expression for the full scalar potential,

$$\begin{aligned} V_{\rm O5/D5} =& \frac{1}{2}\,\rho^{-3}\tau^{-2}\,H_{mnp}\,H_{m'n'p'}\,M^{mm'}\,M^{nn'}\,M^{pp'} + \frac{1}{2}\,\rho\,\tau^{-6}\,F_m\,F_{m'}\,M^{mm'} \\ &+ \frac{\tilde{\lambda}_5^2}{12}\,\tau^{-4}\,g_{IJK}\left[2\,\rho^{-1}H_{mnp}Y^{Im}Y^{Jn}Y^{Kp} + 3\,\rho\,g_{I'J'}{}^K Y^{Im}Y^{Jn}Y^{I'p}Y^{J'q}M_{mn}M_{pq}\right] \\ &+ 2\,\kappa_6^2\,\rho^{-1}\tau^{-4}\,T_{\rm D5}\left(N_{\rm D5} + 2\epsilon_{\rm O5}\right)\,, \end{aligned} \tag{6.98}$$

where the bulk and the WZ contributions correspond to the first line, while the rest come from

DBI.

Matching with $\mathcal{N} = (1,1)$ gauged supergravity

Since the O5 reduction Ansatz respects $\mathrm{SL}(4,\mathbb{R}) \times G_{\mathrm{YM}}$ covariance, the fundamental index M of $\mathrm{SO}(4, 4+\mathfrak{N})$ is split accordingly

$$M \longrightarrow m \oplus \bar{m} \oplus I \ ,$$

where m and $\bar{m}$ are (anti)fundamental indices of $\mathrm{SL}(4,\mathbb{R})$, while I is an adjoint index of G_{YM}. The dictionary between flux parameters and embedding tensor components is summarized in Table 6.3. With these non-vanishing components of the embedding tensor, the general form

IIB fluxes	Θ components	Dictionary
$\overline{H}_{mnp} = \epsilon_{mnpq}\, h^q$	$\zeta_{\bar{m}}$	$\zeta_{\bar{m}} = h^m$
$\overline{F}_m = f_m$	$f_{\bar{m}\bar{n}\bar{p}}$	$f_{\bar{m}\bar{n}\bar{p}} = \epsilon^{mnpq}\, f_q$
$g_{IJ}{}^K$	f_{IJK}	$f_{IJK} = \tilde{\lambda}_5^{-1}\, g_{IJ}{}^L \kappa_{KL}$

Table 6.3: *The embedding tensor/fluxes dictionary for type IIB reductions with spacetime filling O5-planes and D5-branes.*

of the QC (6.72) reduces to the Jacobi identities for the structure constants g_{IJK}, as given in (6.84).

In order to evaluate the general supergravity scalar potential (6.74) in our specific case, we need to identify how the $17 + 4\mathfrak{N}$ scalars parametrize Σ and $\mathcal{H}_{MN}$ appearing there. This is done by directly expressing Σ and the coset representative $\mathcal{V}_M{}^{\underline{M}}$ as a function of the scalars $\big(\rho, \tau, M_{mn}, \gamma^{mn}, Y^{Im}\big)$. In particular, we find that

$$\Sigma = \rho^{-1/2} \ , \tag{6.99}$$

and the $\mathrm{SO}(4, 4+\mathfrak{N})$ coset element is

$$\mathcal{V}_M{}^{\underline{M}} = \left(\begin{array}{c|c|c} \tau^{-1} L_m{}^{\underline{m}} & \mathbb{O}_4 & \mathbb{O}_{4,N} \\ \hline \tau^{-1} \mathcal{C}^{mn} L_n{}^{\underline{m}} & \tau\, L^m{}_{\underline{m}} & -\tilde{\lambda}_5 Y^{Im} \delta_I^{\underline{I}} \\ \hline \tau^{-1} \tilde{\lambda}_5 \delta_{IJ} Y^{Jm} L_m{}^{\underline{m}} & \mathbb{O}_{N,4} & \delta_I^{\underline{I}} \end{array}\right) \ , \tag{6.100}$$

where $L_m{}^{\underline{m}}$ is an $\mathrm{SL}(4,\mathbb{R})/\mathrm{SO}(4)$ coset element satisfying $L_m{}^{\underline{m}} L_n{}^{\underline{m}} \stackrel{!}{=} M_{mn}$, while

$$\mathcal{C}^{mn} \equiv \gamma^{mn} - \frac{\tilde{\lambda}_5^2}{2} Y^{Im} Y^{Jn} \delta_{IJ} \ . \tag{6.101}$$

By plugging the above parametrization of the scalars and embedding tensor into the general form of the scalar potential (6.74), we find a perfect agreement with (6.98), which was calculated from direct dimensional reduction.

6.5 | O7/D7 & Open Strings

Let us now consider reductions of type IIB with the inclusion of spacetime filling O7-planes. These are placed as follows within ten-dimensional spacetime

$$\text{O7}\;:\;\underbrace{\times\;\times\;\times\;\times\;\times\;\times}_{\text{6D spacetime}}\;|\;\underbrace{\underbrace{\times\;\times}_{y^a}\;\underbrace{-\;-}_{y^i}}_{y^m}\;,\qquad \sigma_{\text{O7}}\;:\;y^i\;\longrightarrow\;-y^i\;,$$

where σ_{O7} is the orientifold involution, whose action flips the sign of all transverse coordinates, while leaving the y^a internal coordinates invariant. The O7 projection is realized at the level of the ten-dimensional supergravity fields by means of the simultaneous action of the aforementioned involution, together with the fermionic number $(-1)^{F_L}$ and the worldsheet parity operator. Such a procedure yields the correct field content of a half-maximal supergravity in six dimensions. The resulting details of this are collected in Table 6.4.

IIB fields	σ_{O7}	$(-1)^{F_L}\Omega$	# dof's
$e^a{}_b \oplus e^i{}_j$	$+$	$+$	$2(4-1) = 6$
$e^a{}_j \oplus e^i{}_b$	$-$	$+$	—
B_{ai}	$-$	$-$	4
B_{ab}	$+$	$-$	—
B_{ij}	$+$	$-$	—
Φ	$+$	$+$	1
$C_{(0)}$	$+$	$+$	1
C_{ai}	$-$	$-$	4
C_{ab}	$+$	$-$	—
C_{ij}	$+$	$-$	—
C_{abij}	$+$	$+$	1
Y^{Ii}	$-$	$-$	$2\mathfrak{N}$
$\mathcal{A}^I{}_a$	$+$	$+$	$2\mathfrak{N}$

Table 6.4: *The $\mathbb{Z}_2$ parity of all internal components of the different IIB fields in the presence of spacetime filling O7-planes. The allowed ones yield excitable 6D scalar fields. Note that the total amount of resulting scalars correctly gives* $17 + 4\mathfrak{N}$, i.e. *the dimension of the supergravity coset* (6.68).

Now, because the embedding of O7-planes breaks internal diffeomorphism covariance, the reduction scheme can only be formulated in a $\text{SL}(2,\mathbb{R})_a \times \text{SL}(2,\mathbb{R})_i \times G_{\text{YM}}$ covariant way. The ten-dimensional supergravity Ansatz containing the 17 closed string scalars reads

$$ds^2 = \tau^{-2}\, g_{\mu\nu} dx^\mu dx^\nu \,+\, \rho\,\left(\sigma^2 M_{ab}\, v^a v^b \,+\, \sigma^{-2} M_{ij}\, v^i v^j\right)\;, \tag{6.102}$$

$$e^\Phi = \rho\,\tau^{-2}\;, \tag{6.103}$$

$$B_{(2)} = B_{ai}\, v^a \wedge v^i \,+\, \beta \,+\, \ldots\;, \tag{6.104}$$

$$C_{(0)} = \chi \,+\, \alpha \,+\, \ldots\;, \tag{6.105}$$

$$C_{(2)} = C_{ai}\, v^a \wedge v^i \,+\, C_{(0)}\, B_{ai}\, v^a \wedge v^i \,+\, \gamma \,+\, \dots \,, \tag{6.106}$$

$$C_{(4)} = \frac{\psi}{(2!)^2}\, \epsilon_{ab}\epsilon_{ij}\, v^a \wedge v^b \wedge v^i \wedge v^j \,+\, \dots \,, \tag{6.107}$$

where again "$+\dots$" means that we are discarding the terms in the Ansatz that do not contribute to the scalar potential. The scalars ρ and τ still represent the volume and dilaton would-be moduli, σ is a non-universal geometric deformation controlling the relative size between the two-cycle wrapped by the O7 and the transverse one, M_{ab} and M_{ij} are elements of $\mathrm{SL}(2,\mathbb{R})_{a(i)}/\mathrm{SO}(2)$, and, finally, B_{ai}, C_{ai}, χ, $C_{abij} = \epsilon_{ab}\epsilon_{ij}\psi$ are axionic scalars coming from the NS-NS two-form and the R-R forms. These modes again add up to 17, as expected. Since in this case the metric flux is allowed by the O7 involution, we have introduced a parallelization of the internal manifold with torsion given in terms of the Maurer-Cartan one-forms $v^m = v^m{}_{\underline{m}}\, dy^{\underline{m}}$. These satisfy

$$dv^m \,+\, \frac{1}{2}\,\omega_{np}{}^m v^n \wedge v^p \,=\, 0 \,, \tag{6.108}$$

for some constants $\omega_{np}{}^m$, which turn out to be the structure constants of the underlying Lie algebra and therefore fulfill the Jacobi identities as an integrability condition,

$$\omega_{[mn}{}^r\, \omega_{p]r}{}^q = 0 \,. \tag{6.109}$$

The 0-form $\alpha = \alpha(y)$ entering in the reduction Ansatz of $C_{(0)}$, given in (6.105), introduces the 1-form flux $\overline{F}_a$ via its exterior derivative:

$$d\alpha = \overline{F}_a\, v^a \,, \tag{6.110}$$

with $\overline{F}_a = f_a$. The integrability condition implies that

$$f_a\, \eta_b\, \epsilon^{ab} = 0 \,. \tag{6.111}$$

The 2-form $\beta = \frac{1}{2}\beta_{ab}\, v^a \wedge v^b$ in the reduction Ansatz of $B_{(2)}$ does not give rise to further scalars but rather to fluxes, as it satisfies:

$$d\beta = \frac{1}{2}\,\overline{H}_{abi}\, v^a \wedge v^b \wedge v^i \,, \tag{6.112}$$

where $\overline{H}_{abi} = h_i\, \epsilon_{ab}$ denotes the H-flux. The integrability condition of this equation is trivially satisfied.

Finally, the 2-form $\gamma = \gamma_{ai}\, v^a \wedge v^i$ in (6.106) encodes the 3-form flux $\overline{F}_{abi}$ via its exterior derivative,

$$d\gamma = \frac{1}{2}\overline{F}_{abi}\, v^a \wedge v^b \wedge v^i \,, \tag{6.113}$$

with $\overline{F}_{abi} = \epsilon_{ab}\, f_i$.

If we now add a stack of N_{D7} D7-branes parallel to the O7, these will require the existence of $\mathfrak{N}$ extra vector multiplets, labelled by the index I. In terms of 6D dof's, besides the new vector fields, our resulting gauged supergravity will again have $4\mathfrak{N}$ new scalar modes. Half of them, denoted as $Y^{I\underline{i}}$, correspond to the scalar fields living in the $\mathfrak{N}$ vector supermultiplets, whereas the remaining half come from the internal components of the worldvolume gauge fields $\mathcal{A}^I{}_a$. This way, the set $\left(Y^{Ii}, \mathcal{A}^I{}_a\right)$ exactly parametrizes $4\mathfrak{N}$ independent extra scalar modes. The compactification Ansatze for the worldvolume gauge fields and the scalar fields are

$$\mathcal{A}^I \;=\; \mathcal{A}^I{}_a v_0^a \,+\, \sigma^I \,+ \dots \;, \qquad\qquad Y^{I\underline{i}} \;=\; Y^{Ii}\,(v_0^{-1})_i^{\underline{i}} \;, \tag{6.114}$$

where $v_0^m \;=\; v^m{}_{\underline{m}}|_{y^i=0}\, dy^{\underline{m}}$, with $m \;=\; (a, i)$, are the Maurer-Cartan 1-forms restricted to the worldvolume of the D7 branes[10] and σ^I is a 1-form satisfying

$$g_{IJ}{}^K\,\sigma^I = 0\,, \tag{6.115}$$

where $g_{IJ}{}^K$ are the Yang-Mills structure constants satisfying the Jacobi identities:

$$g_{[IJ}{}^{I'}\, g_{K]I'}{}^L = 0\ . \tag{6.116}$$

In addition, σ^I gives rise to the flux $\overline{\mathcal{F}}^I{}_{ab}$ through

$$d\sigma^I \;=\; \tfrac{1}{2}\overline{\mathcal{F}}^I{}_{ab}\, v_0^a \wedge v_0^b\,, \tag{6.117}$$

where $\overline{\mathcal{F}}^I{}_{ab} = g^I\,\epsilon_{ab}$. The above condition (6.115) on σ^I has to be imposed in order to remove undesired dependence on the internal coordinates. Taking a exterior derivative, we can express this condition in terms of the fluxes g^I as follows:

$$g_{IJ}{}^K\, g^I \;=\; 0\,, \tag{6.118}$$

which tells us that the Killing-Cartan metric,

$$\kappa_{IJ} = g_{IK}{}^L g_{JL}{}^K\,, \tag{6.119}$$

must be degenerate when the fluxes g^I are turned on. Consequently, this implies that the Lie algebra cannot be semisimple. As we are going to see in what follows, the constraint (6.118) will arise in supergravity as one of the QC, (6.133). Let us further remark that the integrability condition of (6.117) is automatically satisfied without imposing further constraints on the fluxes g^I. Such circumstance is very particular of this case. Taking these considerations into account and the compactification Ansatz, we find that the internal components of the field strength $\mathcal{F}^I$ are given by

$$\mathcal{F}^I{}_{ab} = \overline{\mathcal{F}}^I{}_{ab} - g_{JK}{}^I\mathcal{A}^J{}_a\mathcal{A}^K{}_b + 2\mathcal{A}^I{}_{[a}\,\eta_{b]}\,. \tag{6.120}$$

Due to the presence of the O7 plane, the $\mathbb{Z}_2$ truncation realized by the product $\sigma_{\text{O7}}\,(-1)^{F_L}\Omega$ projects out, some internal fluxes. Table 6.5 shows the exhaustive list of fluxes that are projected in by this truncation, where we observe that the fluxes $\omega_{ab}{}^i$, $\omega_{ij}{}^k$, $\omega_{ai}{}^j$ are forbidden. As a

[10] This is nothing but the leading-order contribution in λ of the pull-back of e^a onto the D7 worldvolume, which is all supergravity can capture.

consequence, the Maurer-Cartan 1-forms v^m are required to depend on the internal coordinates y^m in a very particular way. This, together with the consistency of the Ansatz when studying the D7 brane effective action imposes the following functional dependence of the twist matrices $v^m{}_{\underline{m}}$:

$$v^m{}_{\underline{m}} = \begin{pmatrix} v^a{}_{\underline{b}}(y^c) & v^a{}_{\underline{j}}(y^k) \\ \mathbb{O}_2 & v^i{}_{\underline{j}}(y^c) \end{pmatrix} . \tag{6.121}$$

The presence of the D7/O7 sources modifies the Bianchi identity of $C_{(0)}$ as follows,

$$dF_{(1)} = j_{(4)}^{\text{O7/D7}} , \tag{6.122}$$

where $j_{(4)}^{\text{O7/D7}}$ is the effective 7-brane current. The above equation involves both the metric flux $\omega_{ij}{}^a$ and the $F_{(1)}$ flux, $\overline{F}_a$. Upon integration over the transverse space, one obtains the following tadpole condition

$$\frac{1}{2}\epsilon^{ij}\,\omega_{ij}{}^a\overline{F}_a \overset{!}{=} Q_7 , \tag{6.123}$$

where Q_7 is the total charge, receiving contributions both from the D7 branes and the O7,

$$Q_7 = T_{\text{D7}}(N_{\text{D7}} + 8\epsilon_{\text{O7}}) , \tag{6.124}$$

where $\epsilon_{\text{O7}} = \pm 1$ amounts to considering the presence of O7$^{\pm}$ planes. As a consequence of (6.123), we will obtain a non-vanishing contribution in the scalar potential which is proportional to the effective tension, as we are about to see.

The procedure one has to follow to compute the scalar potential is exactly the same as in D5/O5 case studied in the previous section, so we will skip most of the details here. In order to evaluate the contribution from the DBI, we just need to know the matrices $\mathbb{M}$ and $\mathbb{Q}$ in (3.20). In the case at hands, these read

$$\mathbb{M}_{\mu\nu} = \tau^{-2}\,g_{\mu\nu} + \dots , \tag{6.125}$$

$$\mathbb{M}_{ab} = \rho\,\sigma^2 M_{ab} + \lambda\,\mathbb{M}^{(1)I}{}_{ab}\,t_I + \lambda^2\,\rho\,\mathbb{M}^{(2)IJ}{}_{ab}\,t_I\,t_J + \dots , ,$$

$$\sqrt{\det \mathbb{Q}^i{}_j} = 1 + \frac{\lambda^2}{4}\rho^2\sigma^{-4}M_{ij}M_{i'j'}Y^{Ki}Y^{Li'}Y^{K'j}Y^{L'j'}g_{KL}{}^I g_{K'L'}{}^J t_I\,t_J + \dots , \tag{6.126}$$

where

$$\begin{aligned} \mathbb{M}^{(1)I}{}_{ab} &= 2B_{[a|i}Y^{Ij}\omega_{|b]j}{}^i + 2A^J{}_{[a}B_{b]i}Y^{Ki}g_{JK}{}^I \\ &\qquad + Y^{Ii}\,\overline{H}_{abi} - B_{[a|i}B_{|b]j}Y^{Ji}Y^{Kj}g_{JK}{}^I + Y^{Ii}\,\eta_{[a}\,B_{b]i} + \mathcal{A}^I{}_{[a}\,\eta_{b]} + \mathcal{F}^I{}_{ab} , \end{aligned} \tag{6.127}$$

$$\begin{aligned} \mathbb{M}^{(2)IJ}{}_{ab} &= \sigma^2\,M_{c(a}\,Y^{Ii}Y^{Kj}\left(A^L{}_{b)}\,\omega_{ij}{}^c g_{LK}{}^J + \delta^J{}_K\,\omega_{b)j}{}^k\omega_{ki}{}^c\right) \\ &\quad + \sigma^{-2}\,M_{ij}\Big(Y^{Ik}Y^{Jl}\omega_{(a|k}{}^i\omega_{|b)l}{}^j - 2A^K{}_{(a}Y^{Li}Y^{Ik}\omega_{b)k}{}^j g_{KL}{}^J \\ &\qquad + A^K{}_{(a}A^{K'}{}_{b)}Y^{Li}Y^{L'j}g_{KL}{}^I g_{K'L'}{}^J \end{aligned}$$

$$
\begin{aligned}
& - 2B_{(a|k}A^{K}{}_{|b)}Y^{K'k}Y^{Li}Y^{L'j}g_{KL}{}^{I}g_{K'L'}{}^{J} \\
& + B_{(a|k}B_{|b)l}Y^{Kk}Y^{K'l}Y^{Li}Y^{L'j}g_{KL}{}^{I}g_{K'L'}{}^{J} \\
& -\tfrac{1}{4}\,Y^{Ii}Y^{Jk}\left(\eta_a\,\eta_b\,\delta^{j}{}_{k} + 4\eta_{(a}\,(\kappa_{b)})_{k}{}^{j}\right)\right) \ . \qquad (6.128)
\end{aligned}
$$

On the other hand, the bulk contribution was already studied in [52] and has been reviewed in section 6.1. As emphasized in the previous section, we now have to take into account that open strings backreact onto the bulk fields modifying their field strengths as follows,

$$
H_{abi} = \overline{H}_{abi} \,-\, 2\left(\kappa_{[a}\right)_{i}{}^{j}B_{b]j} \,+\, \eta_{[a}B_{b]i} \ , \qquad (6.129)
$$

$$
F_{a} = \overline{F}_{a} + \frac{\tilde{\lambda}_7^2}{2}\left(\left(\kappa_a\right)_{i}{}^{j}\,Y^{Ii}\,Y^{Ik}\,\epsilon_{jk} - g_{IJK}\mathcal{A}^{I}{}_{a}Y^{Ji}Y^{Kj}\,\epsilon_{ij}\right) \ , \qquad (6.130)
$$

$$
\begin{aligned}
F_{abi} = {} & \overline{F}_{abi} - \,2\left(\kappa_{[a}\right)_{i}{}^{j}C_{b]j} \,+\, \eta_{[a}C_{b]i} + 2F_{[a}B_{b]i} - \overline{H}_{abi}\chi \\
& + \tilde{\lambda}_7^2\left[\left(\mathcal{F}^{I}{}_{ab} + \mathcal{A}^{I}{}_{[a}\,\eta_{b]} + \tfrac{1}{2}\overline{H}_{abj}Y^{Ij}\right)\epsilon_{ki} + \mathcal{A}^{I}{}_{[a}\,(\kappa_{b]})_{k}{}^{j}\,\epsilon_{ji}\right]Y^{Ik} \ , \qquad (6.131)
\end{aligned}
$$

where $\tilde{\lambda}_7 \equiv (2\kappa_8^2\,N_{\text{D7}}\,T_{\text{D7}})^{1/2}\,\lambda$ and $g_{IJK} \equiv g_{IJ}{}^{L}\,\kappa_{LK}$. Details on the derivation of these modified field strengths are provided in section 6.2.2.

All these partial results already allow us to compute the scalar potential. It turns out to be given by

$$
\begin{aligned}
V_{\text{O7/D7}} = {} & \frac{1}{4}\rho^{-1}\,\tau^{-2}\,\omega_{mn}{}^{r}\,\omega_{pq}{}^{s}M^{nq}\left(M_{rs}\,M^{mp} + 2\,\delta^{m}{}_{s}\,\delta^{p}{}_{r}\right) + \frac{1}{2}\,\rho\,\tau^{-6}\,\sigma^{-2}\,F_a\,F_b\,M^{ab} \\
& + \frac{1}{4}\,\sigma^{-2}\left(\rho^{-3}\,\tau^{-2}\,H_{abi}\,H_{cdj} + \rho^{-1}\,\tau^{-6}\,F_{abi}\,F_{cdj}\right)\,M^{ac}M^{bd}M^{ij} \\
& + \tau^{-4}\,\sigma^{2}\,(2\kappa_8^2)T_{\text{D7}}(N_{\text{D7}} + 8\,\epsilon_{\text{O7}}) \\
& + \frac{1}{4}\,\tilde{\lambda}_7^2\,\tau^{-4}\sigma^{-2}\,\kappa_{IJ}\left(2\,\sigma^{2}\,\mathbb{M}^{(2)IJ}{}_{ab}\,M^{ab} - \rho^{-2}\,\mathbb{M}^{(1)I}{}_{ab}\,M^{bc}\,\mathbb{M}^{(1)J}{}_{cd}\,M^{da}\right. \\
& \qquad \left. + \rho^{2}M_{ij}M_{i'j'}Y^{Ki}Y^{Li'}Y^{K'j}Y^{L'j'}g_{KL}{}^{I}g_{K'L'}{}^{J}\right) \ . \qquad (6.132)
\end{aligned}
$$

Let us stress three relevant aspects of the potential: (i) as expected from the tadpole condition, the term proportional to the (non-vanishing) effective tension is present, (ii) the last two lines arise from the DBI action of the D7 branes, and (iii) the WZ contributions are entirely encoded in the modified field strengths $F_{(1)}$ and $F_{(3)}$.[11]

Matching with $\mathcal{N} = (1, 1)$ gauged supergravity

Since the O7 reduction Ansatz respects $\text{SL}(2,\mathbb{R})_a \times \text{SL}(2,\mathbb{R})_i \times G_{\text{YM}}$ covariance, the fundamental index M of $\text{SO}(4, 4 + \mathfrak{N})$ is split into

$$
M \;\longrightarrow\; a \;\oplus\; i \;\oplus\; \bar{a} \;\oplus\; \bar{i} \;\oplus\; I \ ,
$$

[11] It is unclear to us the origin of the terms involving η_a in the last lines of (6.127) and (6.128). Nevertheless, as we are interested in configurations with non-Abelian fluxes, one of the quadratic constraints in (6.133) forces $\eta_a = 0$.

where a & $\bar{a}$ (i & $\bar{i}$) are (anti)fundamental indices of $\mathrm{SL}(2,\mathbb{R})_{a(i)}$, while I is an adjoint index of G_{YM}. The dictionary between flux parameters and embedding tensor components is summarized in Table 6.5.

IIB fluxes	Θ components	Dictionary
$\overline{H}_{abi} = \epsilon_{ab}\, h_i$	$f_{ab\bar{i}}$	$f_{ab\bar{i}} = \epsilon_{ab}\epsilon^{ij} h_j$
$\omega_{ai}{}^{j} = (\kappa_a)_i{}^j + \frac{1}{2}\eta_a\delta_i^j$	$f_{ai\bar{j}} \oplus \xi_a$	$f_{ai\bar{j}} = (\kappa_a)_i{}^j$ $\xi_a = -\eta_a$
$\omega_{ab}{}^{c} = -2\eta_{[a}\delta_{b]}^c$	$f_{ab\bar{c}}$	$f_{ab\bar{c}} = \frac{1}{2}\epsilon_{ab}\epsilon^{cd}\eta_d$
$\omega_{ij}{}^{a} = \epsilon_{ij}\theta^a$	ζ_a	$\zeta_a = -\epsilon_{ab}\theta^b$
$\overline{F}_a = f_a$	f_{aij}	$f_{aij} = \epsilon_{ij} f_a$
$\overline{F}_{abi} = \epsilon_{ab} f_i$	f_{abi}	$f_{abi} = \epsilon_{ab} f_i$
$\overline{\mathcal{F}}^I{}_{ab} = \epsilon_{ab} g^I$	f_{abI}	$f_{abI} = \tilde{\lambda}_7\epsilon_{ab}\delta_{IJ} g^J$
$g_{IJ}{}^K$	f_{IJK}	$f_{IJK} = -\tilde{\lambda}_7^{-1}\, g_{IJ}{}^L\, \kappa_{LK}$

Table 6.5: *The embedding tensor/fluxes dictionary for type IIB reductions with spacetime filling O7-planes and D7-branes. Note that κ_a satisfies $(\kappa_a)_i{}^i = 0$, and η_a parametrizes the partial traces of ω, while still respecting $\omega_{mn}{}^n = 0$, which is required by unimodularity.*

With these non-vanishing components of the embedding tensor, the general form of the QC (6.72) reduces to

$$\begin{aligned} \epsilon^{ab}\left(\eta_a (\kappa_b)_i{}^j - (\kappa_a)_i{}^k (\kappa_b)_k{}^j\right) &= 0 \ , & \epsilon^{ab} f_a \eta_b &= 0 \ , \\ \eta_a\, g_{IJK} &= 0 \ , & g^I g_{IJK} &= 0 \ , \\ g_{[IJ}{}^{I'}\, g_{K]LI'} &= 0 \ , & \eta_a\theta^b - \tfrac{1}{2}\,\delta_a^b\eta_c\theta^c &= 0 \ , \end{aligned} \tag{6.133}$$

which exactly reproduce the consistency constraints coming from both the Bianchi and Jacobi identities of the corresponding flux background. In particular, the first and last quadratic constraints arise from the Jacobi identities of the structure constants (6.109) associated to the group manifold.

For the evaluation of the general supergravity scalar potential (6.74) in this specific case, we again need to identify how Σ and the coset representative $\mathcal{V}_M{}^{\underline{M}}$ are expressed as a function of the $17 + 4\mathfrak{N}$ scalars $\left(\rho, \tau, \sigma, M_{ab}, M_{ij}, B_{ai}, C_{ai}, \chi, \psi, \mathcal{A}^I{}_a, Y^{Ii}\right)$. We find

$$\Sigma = \sigma \ , \tag{6.134}$$

while

$$\mathcal{V}_M{}^{\underline{M}} = \left(\begin{array}{c|c|c} \tau L_m{}^{\underline{m}} & \tau^{-1}\mathcal{C}_{mn}L^n{}_{\underline{m}} & -A^I{}_m\delta_I^{\underline{I}} \\ \hline \mathbb{O}_4 & \tau^{-1}\, L^m{}_{\underline{m}} & \mathbb{O}_{4,N} \\ \hline \mathbb{O}_{N,4} & \tau^{-1}\delta_{IJ}A^J{}_n L^n{}_{\underline{m}} & \delta_I^{\underline{I}} \end{array}\right) \ , \tag{6.135}$$

where

$$L^m{}_{\underline{m}} \equiv \left(\begin{array}{c|c} \rho^{-1/2}\ell^a{}_{\underline{a}} & \mathbb{O}_2 \\ \hline -\rho^{-1/2}B_{bj}\epsilon^{ji}\ell^b{}_{\underline{a}} & \rho^{1/2}\ell^i{}_{\underline{i}} \end{array}\right) \ , \tag{6.136}$$

with $\ell^a{}_{\underline{a}}$ & $\ell^i{}_{\underline{i}}$ satisfying $\ell^a{}_{\underline{a}}\ell^b{}_{\underline{a}} = M^{ab}$ and $\ell^i{}_{\underline{i}}\ell^j{}_{\underline{i}} = M^{ij}$, respectively. The matrix $\mathcal{C}$ is defined as $\mathcal{C}_{mn} \equiv \gamma_{mn} - \frac{1}{2} A^I{}_m A^I{}_n$, in terms of

$$\gamma_{mn} \equiv \left(\begin{array}{cc|c} 0 & \psi & \\ -\psi & 0 & -C_{aj} \\ \hline & C_{bi} & \begin{array}{cc} 0 & -\chi \\ \chi & 0 \end{array} \end{array} \right) \quad , \qquad A^I{}_m \equiv \tilde{\lambda}_7 \left(\, \mathcal{A}^I{}_a \; \middle| \; Y^{Ij} \epsilon_{ji} \, \right) \quad . \tag{6.137}$$

Thus, plugging $\mathcal{V}_M{}^{\underline{M}}$ together with the parametrization of the embedding tensor $\Theta \equiv \{ f_{MNP}, \xi_M, \zeta_M \}$ in the gauged supergravity potential (6.74), we obtain the same scalar potential as the one calculated from the compactification in (6.132).

6.6 | O9/D9 & Open Strings

Let us finally study type IIB reductions with spacetime filling O9-planes, *i.e.* type I reductions. The orientifold planes in this case fill the entire ten-dimensional spacetime:

$$\text{O9} \;:\; \underbrace{\times\;\times\;\times\;\times\;\times\;\times}_{\text{6D spacetime}} \,\big|\, \underbrace{\times\;\times\;\times\;\times}_{y^m} \,,$$

where σ_{O9} acts trivially on all the coordinates, due to the absence of transverse directions. In this case the $\mathbb{Z}_2$ action realizing the truncation is purely given by the worldsheet parity operator Ω. The set of resulting scalar modes retained by this operation is shown in Table 6.6.

IIB fields	σ_{O9}	Ω	# dof's
$e^m{}_n$	$+$	$+$	$16 - 6 = 10$
B_{mn}	$+$	$-$	—
Φ	$+$	$+$	1
$C_{(0)}$	$+$	$-$	—
C_{mn}	$+$	$+$	6
C_{mnpq}	$+$	$-$	—
$\mathcal{A}^I{}_m$	$+$	$+$	$4\mathfrak{N}$

Table 6.6: *The $\mathbb{Z}_2$ parity of all internal components of the different IIB fields in the presence of spacetime filling O9-planes. The allowed ones yield excitable 6D scalar fields. Note that the total amount of resulting scalars correctly gives* $17 + 4\mathfrak{N}$, i.e. *the dimension of the supergravity coset* (6.68).

The spacetime filling O9-planes preserve the internal diffeomorphism covariance, thus making the reduction scheme to be formulated in a $\text{SL}(4,\mathbb{R}) \times G_{\text{YM}}$ covariant way. The ten-dimensional supergravity Ansatz that contains the 17 closed string scalars coincides with the one studied by Kaloper and Myers in [113] except for the fact that they work in the heterotic frame and focus on the Abelian case.[12] In terms of the type I fields, our Ansatz reads

$$\begin{aligned} ds^2_{(10)} &= \tau^{-2}\, g_{\mu\nu} dx^\mu dx^\nu \;+\; \rho\, M_{mn}\, v^m v^n + \dots \;, \\ e^\Phi &= \rho\, \tau^{-2} \;, \\ C_{(2)} &= \frac{1}{2}\, C_{mn}\, v^m \wedge v^n - \frac{\tilde{\lambda}_9^2}{2} \mathcal{A}^I \wedge \sigma^I + \gamma + \dots \;, \end{aligned} \tag{6.138}$$

where $\tilde{\lambda}_9 \equiv (2\kappa_{10}^2\, N_{\text{D9}}\, T_{\text{D9}} \lambda^2)^{1/2}$. Once again, the scalars ρ and τ represent the volume and dilaton would-be moduli, M_{mn} is an element of $\text{SL}(4,\mathbb{R})/\text{SO}(4)$, and C_{mn} parametrizes the axionic scalars coming from the R-R 2-form $C_{(2)}$. As expected, these modes add up to 17. As before, we have introduced a parallelization of the internal manifold with torsion given in terms of the Maurer-Cartan one-forms v^m, which fulfill the equation

$$dv^m \;+\; \frac{1}{2}\, \omega_{np}{}^m v^n \wedge v^p \;=\; 0 \;. \tag{6.139}$$

In addition to these fields, when adding a stack of N_{D9} D9-branes parallel to the O9, we will require $\mathfrak{N}$ extra vector multiplets, labelled by the index I to accommodate the full group G_{YM}. In terms of 6D degrees of freedom, besides the new vector fields, our resulting gauged supergravity description will again have $4\mathfrak{N}$ new scalar modes arising from the internal components of the worldvolume gauge fields

$$\mathcal{A}^I \;=\; \mathcal{A}^I{}_m v^m + \sigma^I + \dots \;. \tag{6.140}$$

The 1- and 2-form $\sigma^I(y)$ and $\gamma(y)$ in the Ansatze (6.138) and (6.140) give rise to the vector flux $\overline{\mathcal{F}}^I{}_{mn}$ and the 3-form flux $\overline{F}_{mnp}$ listed in Table 6.7 via their exterior derivatives [113],

$$d\sigma^I - \frac{1}{2} \overline{\mathcal{F}}^I{}_{mn} v^m \wedge v^n \;=\; 0\,, \tag{6.141}$$

$$d\gamma - \frac{\tilde{\lambda}_9^2}{2} \sigma^I \wedge d\sigma^I \;=\; \frac{1}{3!} \overline{F}_{mnp}\, v^m \wedge v^n \wedge v^p \,. \tag{6.142}$$

The integrability conditions associated to the above equations are,

$$\overline{\mathcal{F}}^I{}_{q[m}\, \omega_{np]}{}^q \;=\; 0\,, \tag{6.143}$$

$$\frac{\tilde{\lambda}_9^2}{2}\, \overline{\mathcal{F}}^I{}_{[mn} \overline{\mathcal{F}}^I{}_{pq]} - \overline{F}_{r[mn}\, \omega_{pq]}{}^r \;=\; 0\,. \tag{6.144}$$

In order to compute the contribution to the scalar potential coming from the DBI action (3.20), we just need to know explicitly the matrix $\mathbb{M}_{MN}$ (3.21) since the lack of transverse coordinates forbids the existence of $\mathbb{Q}$. The non-trivial components of $\mathbb{M}_{MN}$ are

$$\mathbb{M}_{\mu\nu} = \tau^{-2}\, g_{\mu\nu} \;+\; \dots \;, \tag{6.145}$$

$$\mathbb{M}_{mn} = \rho\, M_{mn} + \lambda\, \mathcal{F}_{mn} \;+\; \dots \;. \tag{6.146}$$

[12]The compactification of the heterotic string effective action on a torus including the non-Abelian sector has been done in [114].

Instead, the bulk contribution can be extracted from using the results provided in section 6.1, but we need to compute first F_{mnp}. To this aim, one has to bear in mind that the coupling of the sources to $C_{(6)}$ modifies the Bianchi identity of $F_{(3)}$ as in (6.59), which we repeat here for convenience

$$dF_{(3)} = -\frac{\tilde{\lambda}_9^2}{2}\mathcal{F}^I \wedge \mathcal{F}_I \,. \tag{6.147}$$

This implies that, locally, the field strength $F_{(3)}$ is given by

$$F_{(3)} = dC_{(2)} - \frac{\tilde{\lambda}_9^2}{2}\left(\mathcal{F}^I \wedge \mathcal{A}^I + \frac{1}{3!}\,g_{IJK}\,\mathcal{A}^I \wedge \mathcal{A}^J \wedge \mathcal{A}^K\right) . \tag{6.148}$$

Restricting to the internal components components and making use of the reduction Ansatz, (6.138) and (6.140), we obtain that

$$F_{mnp} = \overline{F}_{mnp} - 3\,C_{q[m}\,\omega_{np]}{}^q - 3\,\tilde{\lambda}_9^2\,\mathcal{A}^I{}_{[m|}\left(\overline{\mathcal{F}}^I{}_{|np]} - \frac{1}{3}\,g_{IJK}\mathcal{A}^J{}_{|n}\mathcal{A}^K{}_{p]} - \frac{1}{2}\mathcal{A}^I{}_q\,\omega_{|np]}{}^q\right) . \tag{6.149}$$

Finally, the complete expression for the scalar potential is,

$$\begin{aligned} V_{\mathrm{O9/D9}} = {}& \frac{1}{4}\rho^{-1}\tau^{-2}\left(M_{mn}M^{pq}M^{rs}\omega_{pr}{}^m\omega_{qs}{}^n + 2M^{mn}\omega_{mq}{}^p\omega_{np}{}^q\right) \\ & + \frac{\tilde{\lambda}_9^2}{4}\rho^{-1}\tau^{-4}M^{mm'}M^{nn'}\mathcal{F}^I{}_{mn}\mathcal{F}^I{}_{m'n'} + \frac{1}{12}\rho^{-1}\tau^{-6}M^{mm'}M^{nn'}M^{pp'}F_{mnp}F_{m'n'p'} \\ & + 2\,\kappa_6^2\,T_{\mathrm{D9}}(N_{\mathrm{D9}} + 32\,\epsilon_{\mathrm{O9}})\rho\tau^{-4} \;, \end{aligned} \tag{6.150}$$

where

$$\mathcal{F}^I{}_{mn} \equiv \overline{\mathcal{F}}^I{}_{mn} - g_{JK}{}^I\mathcal{A}^J{}_m\mathcal{A}^K{}_n - \mathcal{A}^I{}_p\,\omega_{mn}{}^p \;. \tag{6.151}$$

As a tadpole cannot be generated with the fluxes at our disposal, the condition $N_{\mathrm{D9}}+32\,\epsilon_{\mathrm{O9}} = 0$ must be imposed. This implies $N_{\mathrm{D9}} = 32$ and $\epsilon_{\mathrm{O9}} = -1$, so that the total charge and tension vanish. Therefore, the last term in the scalar potential, which is proportional to the effective tension, vanishes as well.

Matching with $\mathcal{N} = (1, 1)$ gauged supergravity

The O9 reduction Ansatz preserves $\mathrm{SL}(4,\mathbb{R}) \times G_{\mathrm{YM}}$ covariance, in such a way that the fundamental index M of $\mathrm{SO}(4, 4 + \mathfrak{N})$ is split into

$$M \;\longrightarrow\; m \;\oplus\; \bar{m} \;\oplus\; I \;\;,$$

where m & $\bar{m}$ are (anti)fundamental indices of $\mathrm{SL}(4,\mathbb{R})$, and I is an adjoint index of G_{YM}. The dictionary between flux parameters and embedding tensor components is summarized in Table 6.7.

With these non-vanishing components of the embedding tensor, the general form of the QC

IIB fluxes	Θ components	Dictionary
$\overline{F}_{mnp}$	f_{mnp}	$f_{mnp} = -\overline{F}_{mnp}$
$\omega_{mn}{}^{p}$	$f_{mn\bar{p}}$	$f_{mn\bar{p}} = \omega_{mn}{}^{p}$
$\overline{\mathcal{F}}^{I}{}_{mn}$	f_{mnI}	$f_{mnI} = -\tilde{\lambda}_9\,\kappa_{IJ}\,\overline{\mathcal{F}}^{J}{}_{mn}$
$g_{IJ}{}^{K}$	f_{IJK}	$f_{IJK} = \tilde{\lambda}_9^{-1}\,g_{IJ}{}^{L}\kappa_{LK}$

Table 6.7: *The embedding tensor/fluxes dictionary for type IIB reductions with spacetime filling O9-planes and D9-branes. Note that $\omega_{mn}{}^{p}$ is restricted to satisfy $\omega_{mn}{}^{n} = 0$, which is required for unimodular gaugings.*

(6.72) reduces to

$$
\begin{aligned}
-\overline{F}_{r[mn}\,\omega_{pq]}{}^{r} + \frac{\tilde{\lambda}_9^2}{2}\,\overline{\mathcal{F}}^{I}{}_{[mn}\,\overline{\mathcal{F}}^{I}{}_{pq]} \;&=\; 0 \quad , \qquad & \omega_{[mn}{}^{r}\,\omega_{p]r}{}^{q} \;&=\; 0 \quad , \\
\omega_{[mn}{}^{q}\,\overline{\mathcal{F}}^{I}{}_{p]q} \;&=\; 0 \quad , \qquad & g_{IJK}\,\overline{\mathcal{F}}^{K}{}_{mn} \;&=\; 0 \quad , \\
g_{[IJ}{}^{I'}\,g_{K]LI'} \;&=\; 0 \quad ,
\end{aligned}
\tag{6.152}
$$

which exactly reproduce the consistency constraints coming from the BI of the corresponding flux background (see section 6.2.3 for details).

For the evaluation of the general supergravity scalar potential (6.74) in this specific case, we again need to identify how Σ and the coset representative $\mathcal{V}_{M}{}^{\underline{M}}$ are expressed as a function of the $17 + 4\mathfrak{N}$ scalars. In this case, because no fluxes are embedded into ζ_M, the scalar potential is entirely written in terms of $\mathcal{H}_{MN}$. We find then

$$
\Sigma \;=\; \rho^{1/2}\ , \qquad\qquad \Lambda \;=\; \tau^2\ , \tag{6.153}
$$

while

$$
\mathcal{H}_{MN} \;=\; \begin{pmatrix} \tilde{M}_{mn} + \mathcal{C}_{pm}\,\mathcal{C}_{qn}\,\tilde{M}^{pq} + \kappa_{IJ}\,A^{I}{}_{m}\,A^{J}{}_{n} & -\tilde{M}^{np}\,\mathcal{C}_{pm} & A^{J}{}_{m} + \mathcal{C}_{pm}\,\tilde{M}^{pq}\,A^{J}{}_{q} \\ -\tilde{M}^{mp}\,\mathcal{C}_{pn} & \tilde{M}^{mn} & -\tilde{M}^{mp}\,A^{J}{}_{p} \\ A^{I}{}_{n} + \mathcal{C}_{pn}\,\tilde{M}^{pq}\,A^{I}{}_{q} & -\tilde{M}^{np}\,A^{I}{}_{p} & \kappa_{IJ} + A^{I}{}_{p}\,A^{J}{}_{q}\,\tilde{M}^{pq} \end{pmatrix}\ , \tag{6.154}
$$

where

$$
\tilde{M}_{mn} \equiv \Lambda\,M_{mn}\ , \qquad \mathcal{C}_{mn} \equiv\; C_{mn} + \frac{1}{2}\kappa_{IJ}\,A^{I}{}_{m}\,A^{J}{}_{n}\ , \qquad A^{I}{}_{m} \equiv \tilde{\lambda}_9\,\mathcal{A}^{I}{}_{m}\ . \tag{6.155}
$$

Chapter 7

Conclusions

The central theme of this thesis has been the exploration of the pivotal role played by D-branes in two complementary corners of string theory: the study of holography and the dynamics of flux compactifications. Both lines of research arise naturally from the broader quest to understand the non-perturbative structure of string theory, and both highlight the extent to which D-branes serve as the essential building blocks of modern string-theoretic constructions.

On the holographic side, we have investigated the problem of constructing explicit brane realizations for AdS vacua, a crucial ingredient in sharpening the AdS/CFT correspondence. While the pure spinor formalism has enabled impressive progress in classifying supersymmetric AdS backgrounds, the lack of a clear brane interpretation for most vacua has long been a limitation. By analyzing specific brane intersections in type IIB string theory, we provided new examples in which the supergravity solutions admit AdS_3 near-horizon geometries with controlled supersymmetry. These constructions not only yield explicit realizations of conformal defects in strongly coupled gauge theories but also broaden the range of holographic correspondences accessible through brane engineering. In this way, our work contributes to the ongoing effort of grounding holography in explicit string-theoretic setups, where the correspondence can be tested with quantitative precision.

On the compactification side, we have turned our attention to the interplay between fluxes, Dp-branes, and orientifold Op-planes. Flux compactifications have long been recognized as the primary mechanism for moduli stabilization, but the inclusion of open-string degrees of freedom substantially enriches the structure of the resulting low-energy effective theories.

We have studied type IIB flux compactifications down to six dimensions with spacetime filling O-planes, D-branes and open strings. Such compactifications turn out to yield 6D $\mathcal{N} = (1,1)$ gauged supergravities. The exact relation between the 10D and the 6D descriptions was studied in [52] within the closed string sector. Now we have been able to generalize this correspondence to the case where open strings are excited. We also included open string effects such as non-Abelian D-brane gauge groups and non-trivial YM flux. Our analysis is very much in the spirit of the one in [115] carried out in the context of compactifications down to four dimensions, the main difference being that our brane gauge groups may be non-Abelian.

The dictionary derived here allowed us to understand crucial physical mechanisms like the bulk field strength modification induced by open string effects. At least at a heuristic level, this can be related to the GS mechanism for the heterotic string via a duality chain. Besides this intuition, we were able to explain such a form of the bulk field strengths directly in terms of couplings contained in the WZ brane actions.

The work done in this thesis sets the ground for interesting developments within the context of compactifications of type IIB string theory with sixteen supercharges. This will first of all, allow us to search for string vacua supported by interactions between the open and the closed string sectors. This moves towards the direction of [116, 117, 118], where a similar analysis was performed in type IIA strings and conditions for the existence of 4D vacua with *mobile* D-branes are discussed. Our machinery could be extremely valuable in improving efficiency for vacua searches.

Moreover, the possible existence of certain types of vacua within this setup could shed a light on issues of utmost importance, like the validity of Swampland conjectures or the string universality principle. To this end, it would be very interesting to explore the set of non-supersymmetric vacua of this sort (AdS or dS, even), or to study those consistency conditions for flux backgrounds coming from anomaly cancellation. Finally, if one could find supersymmetric AdS extrema in this class, it would be interesting to study their holographic interpretation. We certainly intend to address all of these issues in the next future.

Taken together, the results presented in this thesis underscore the unifying role of D-branes across different regimes of string theory. In holography, they provide the microscopic foundation of the AdS/CFT correspondence; in flux compactifications, they supply the open-string sector necessary for constructing richer and more realistic low-energy models. Beyond their specific technical contributions, these two lines of research reflect a broader theme: progress in string theory often comes from identifying the correct organizing principles –be it through dualities, gauged supergravity formalisms, or explicit brane setups– that bridge the gap between abstract consistency requirements and concrete realizations.

Looking ahead, several avenues remain open. On the holographic side, extending the brane constructions to vacua with less supersymmetry or to non-AdS geometries would provide a deeper understanding of the correspondence beyond its most symmetric incarnations. On the flux compactification side, incorporating higher-order α' corrections and exploring their interplay with open-string fluxes may yield new insights into moduli stabilization and the Swampland constraints. Both directions speak to the same overarching question: how string theory organizes and constrains the vast landscape of consistent quantum gravities.

In conclusion, the investigations carried out in this thesis contribute to clarifying the structure of string vacua and their effective descriptions, while also reinforcing the central position of D-branes in the quest for a unified understanding of quantum gravity.

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